

Les preuves sans mots

Metz 2012

Journées nationales APMEP

30 octobre 2012

Xavier Viennot

avec

Gérard H.E. Duchamp (violon)

animation au violon

identité de Touchard

$$C_{n+1} = \sum_{0 \leq i \leq \lfloor n/2 \rfloor} \binom{n}{2i} C_i 2^{2n-i}$$

C_i



choix d'un chemin de Dyck
de longueur $2i=6$

$$\binom{n}{z_i}$$



choix d'une partie à z_i éléments
parmi $n=10$ éléments

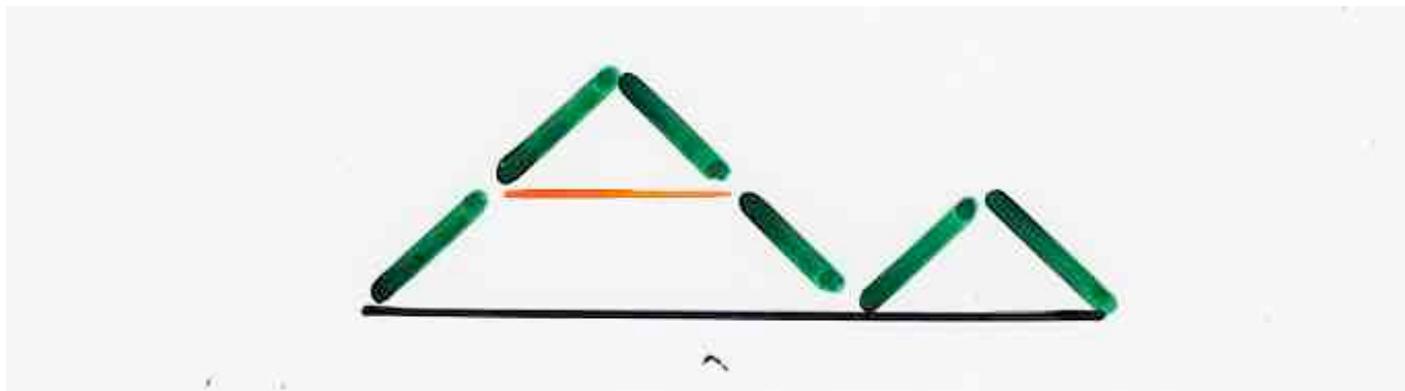
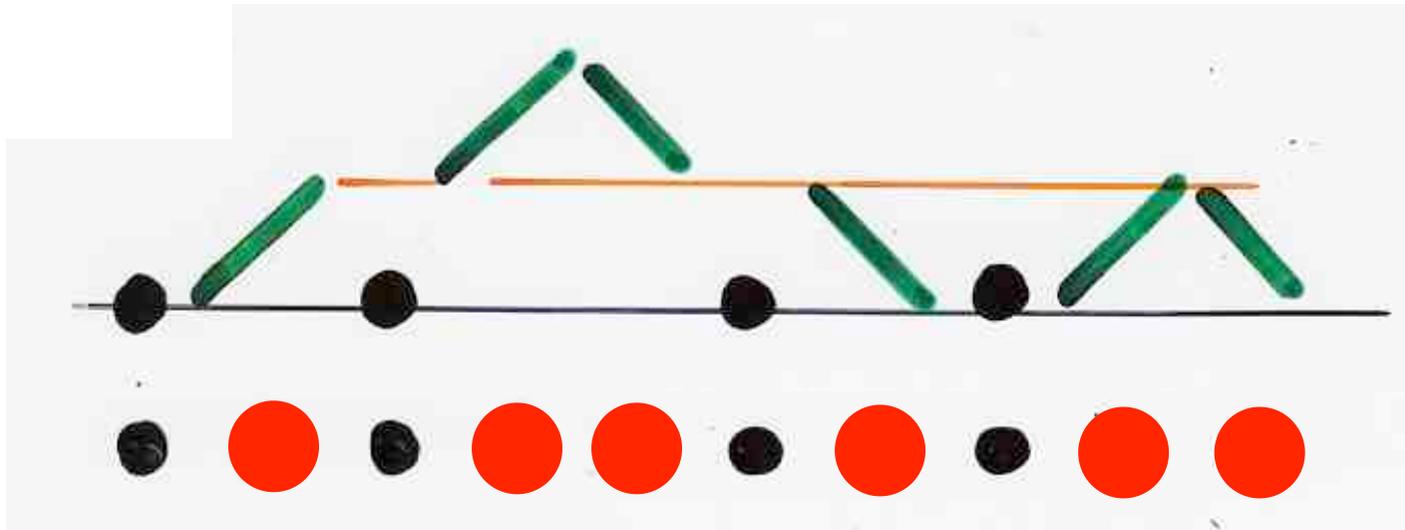
$$\binom{n}{z_i}$$



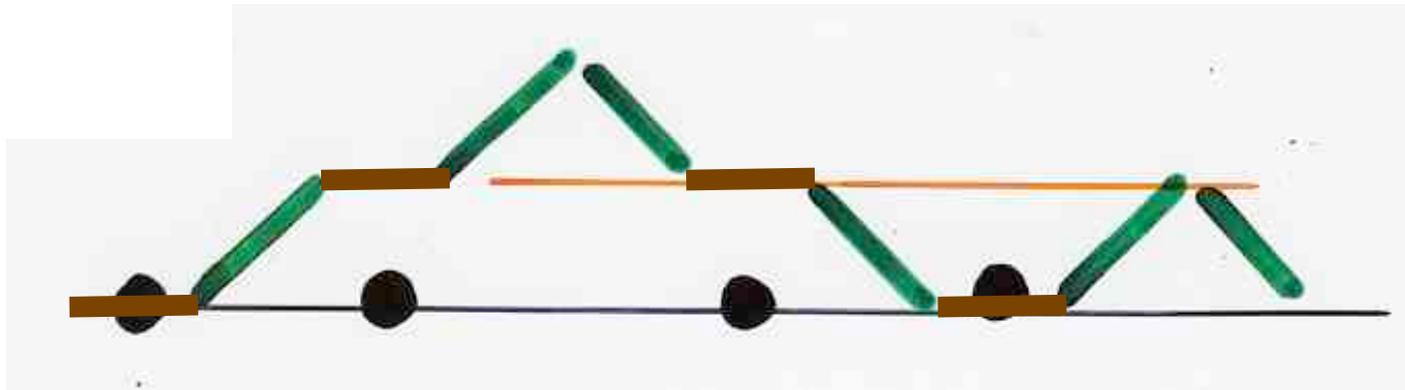
choix d'une partie à z_i éléments
parmi $n=10$ éléments

$$\binom{n}{2i}$$

$$C_i$$



placement du chemin de Dyck sur la partie

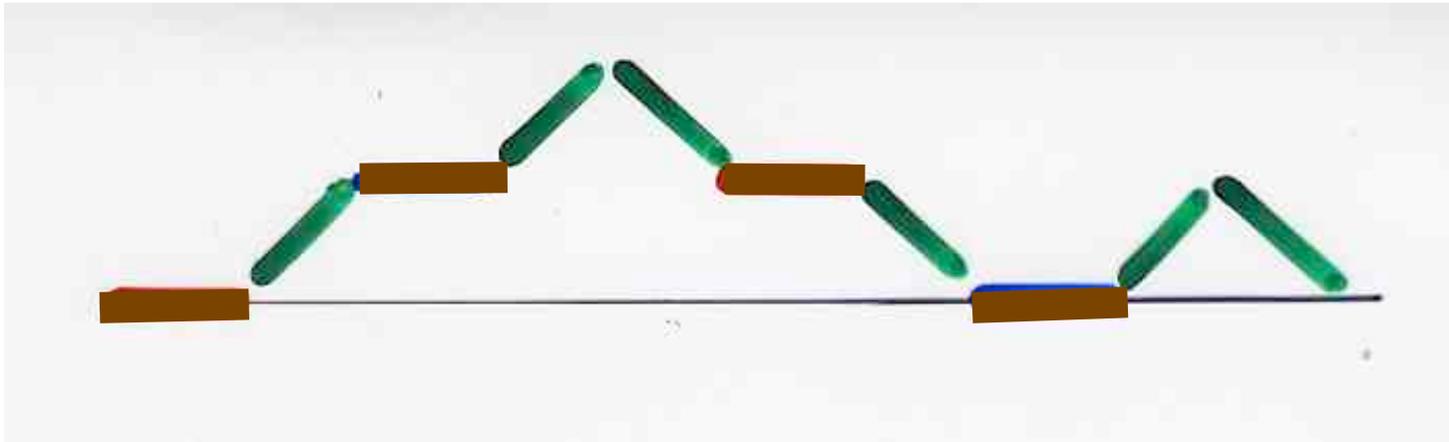


mise en place des paliers

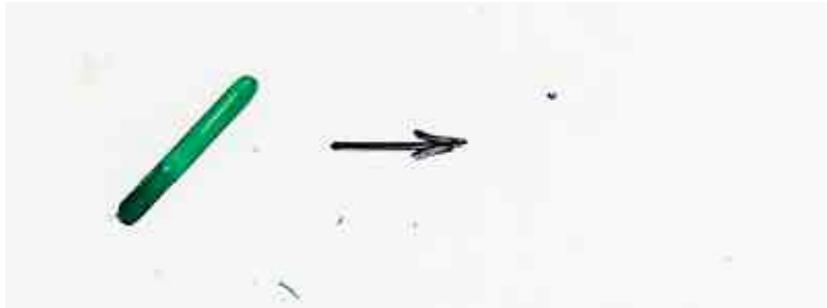


deux couleurs pour les paliers

$$2^{2n-i}$$



colonier tous les paliers



substitution



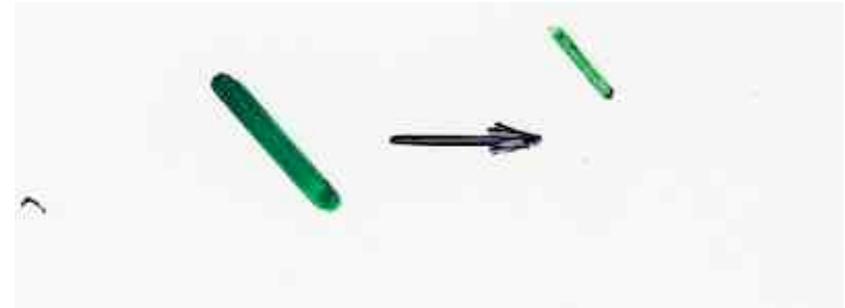
substitution



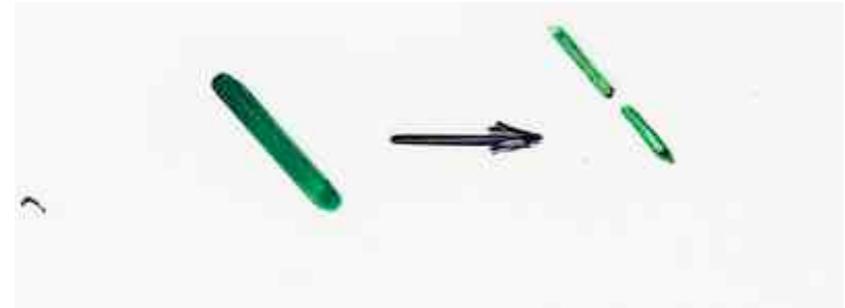
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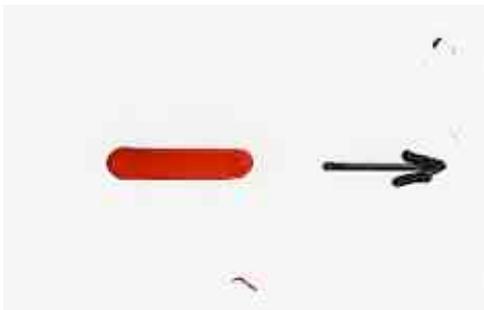
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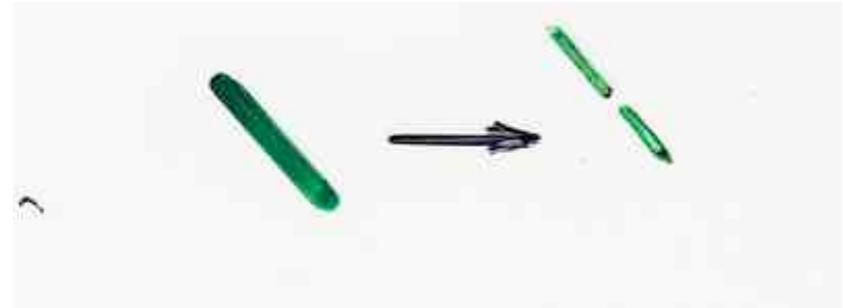
substitution



substitution



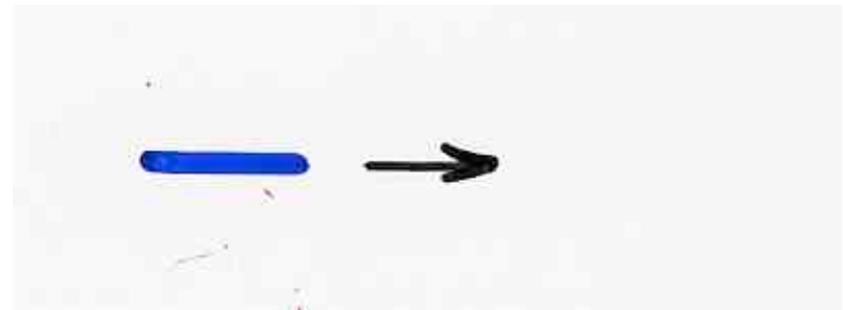
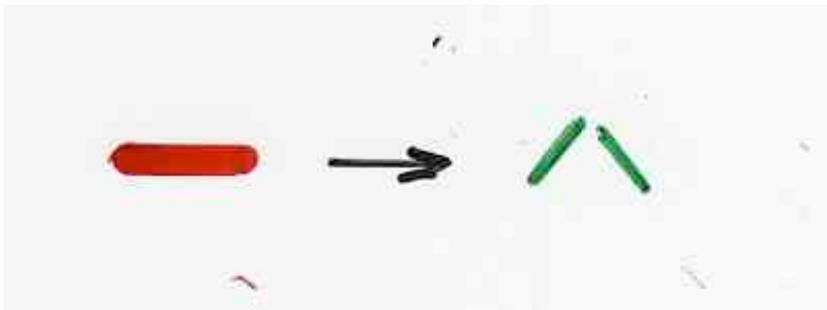
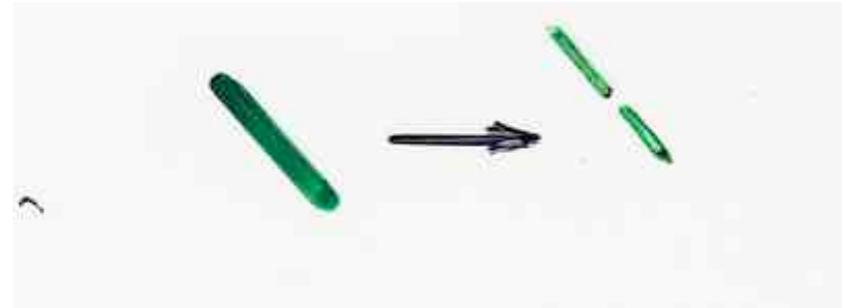
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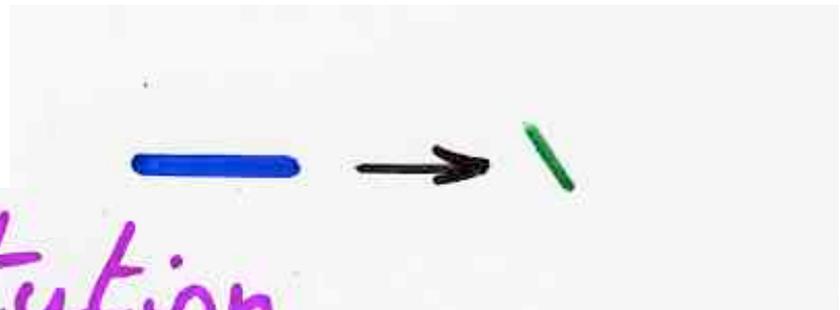
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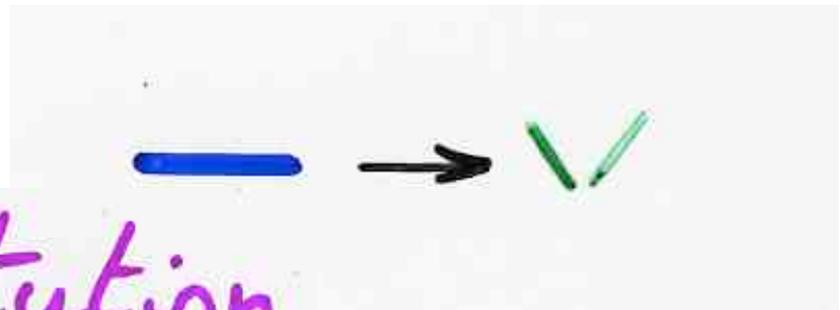
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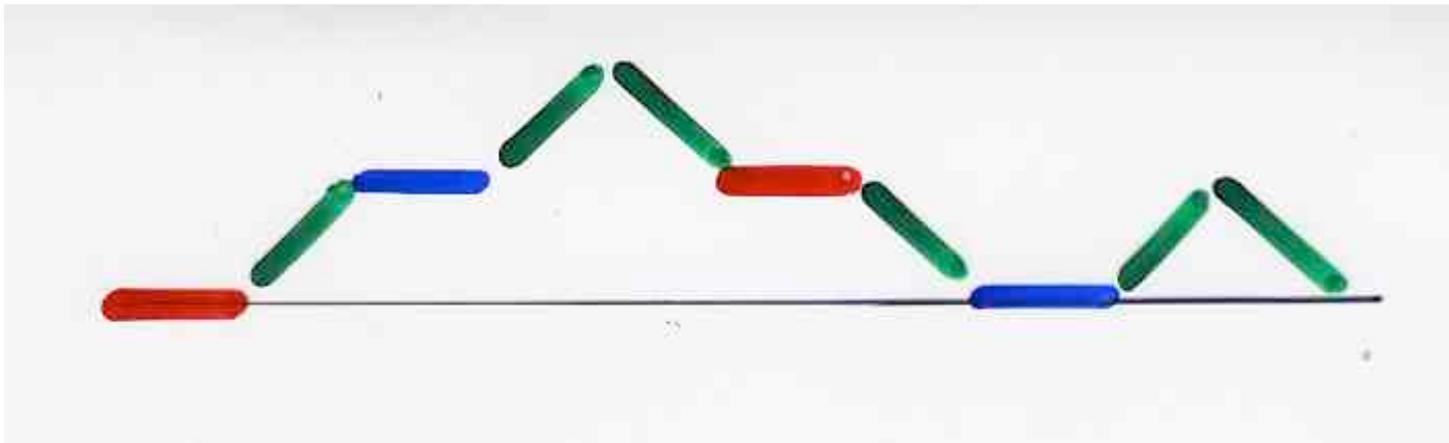
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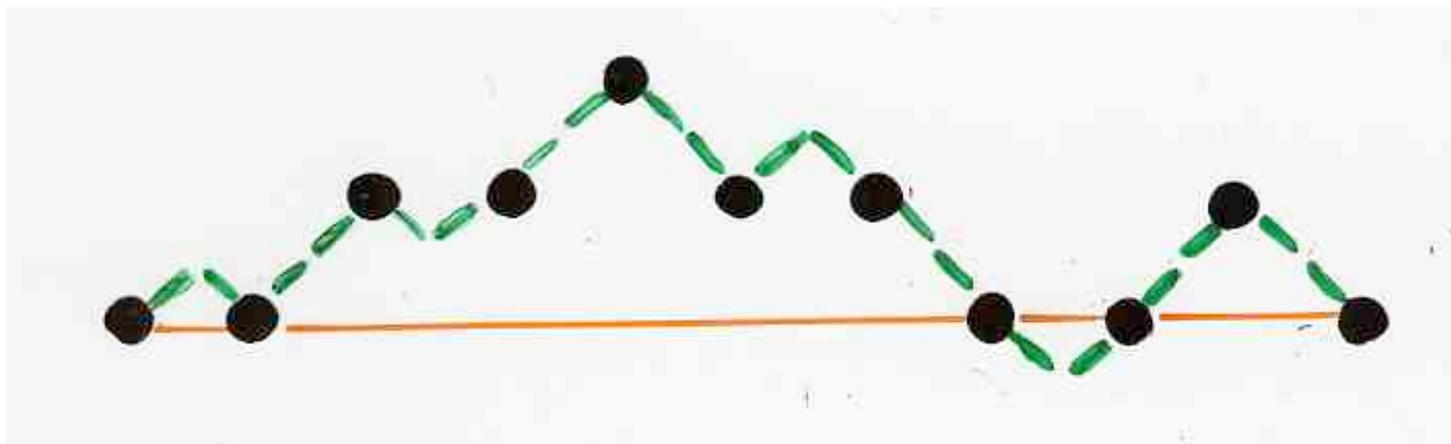
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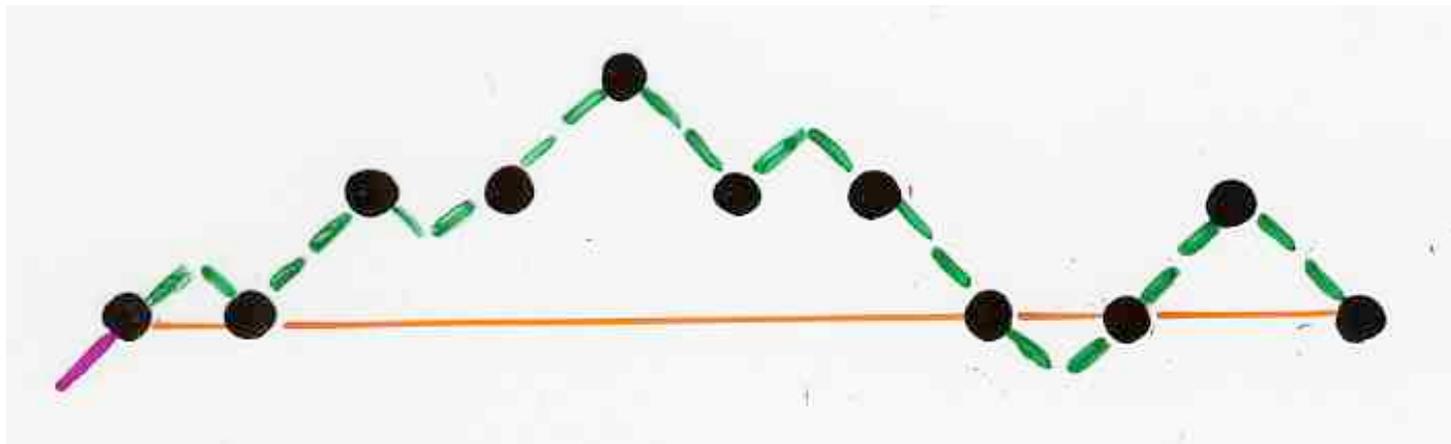


substitution

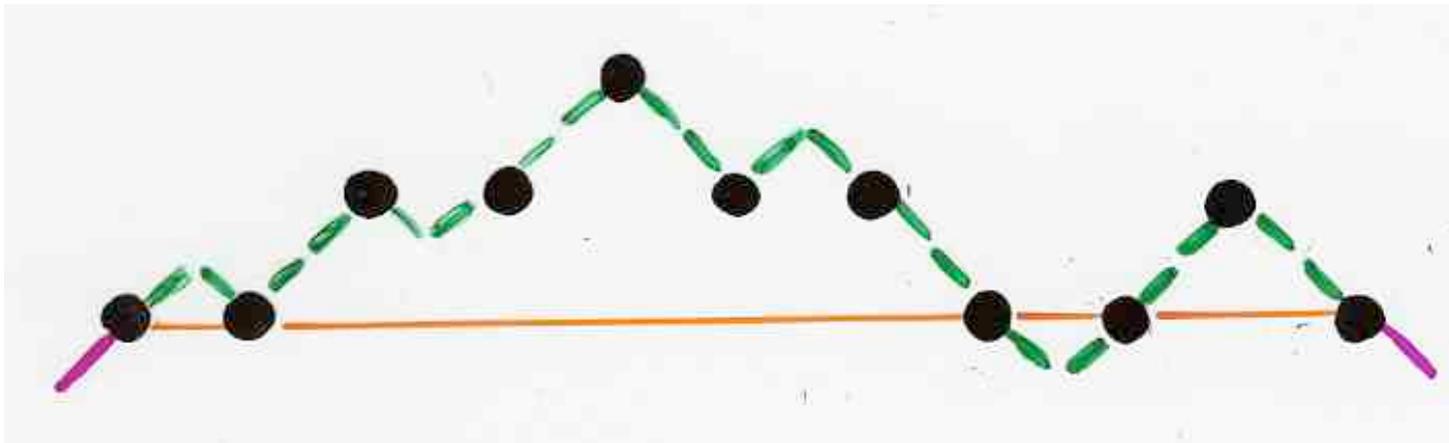


substitution sur tout le chemin



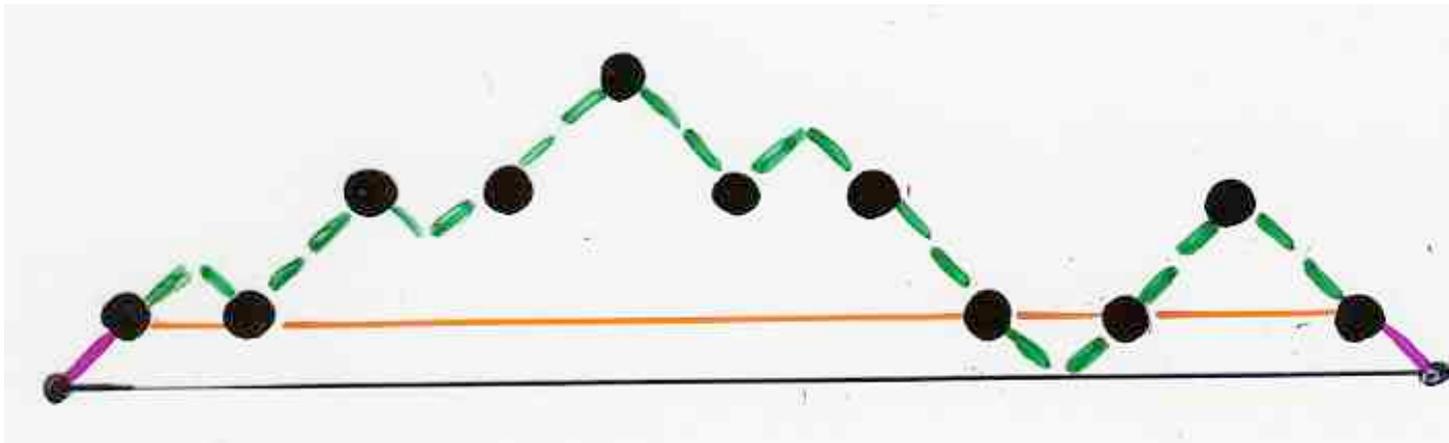


ajout au début
du chemin



ajout à la fin
du chemin

C_{n+1}



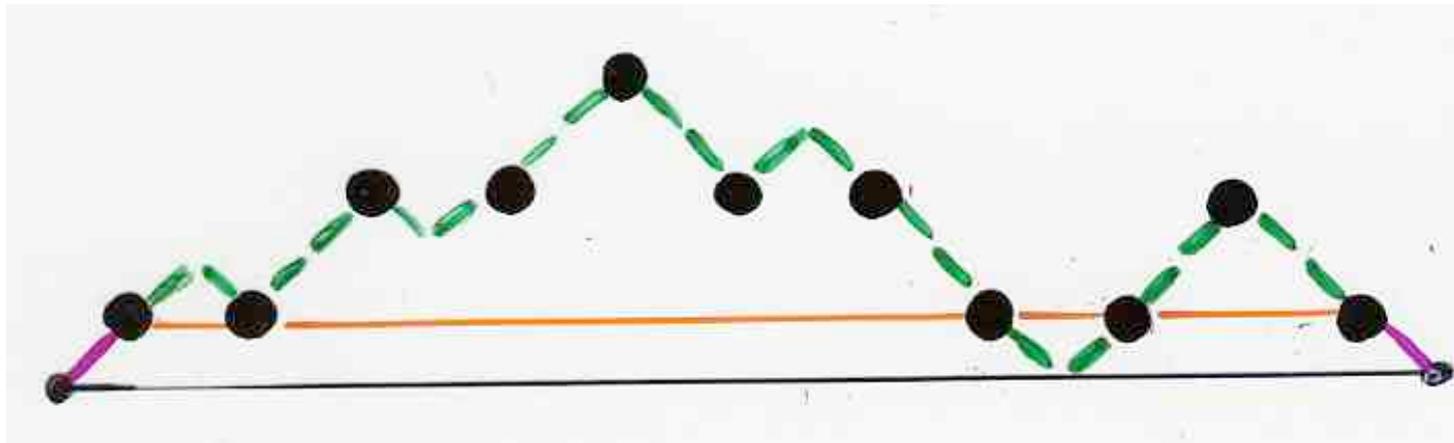
chemin de Dyck de longueur $2i+2$

$$C_{n+1}$$

$$\binom{n}{2i}$$

$$C_i$$

$$2^{2n-i}$$



chemin de Dyck de longueur $2i+2$

bijection
réciproque

