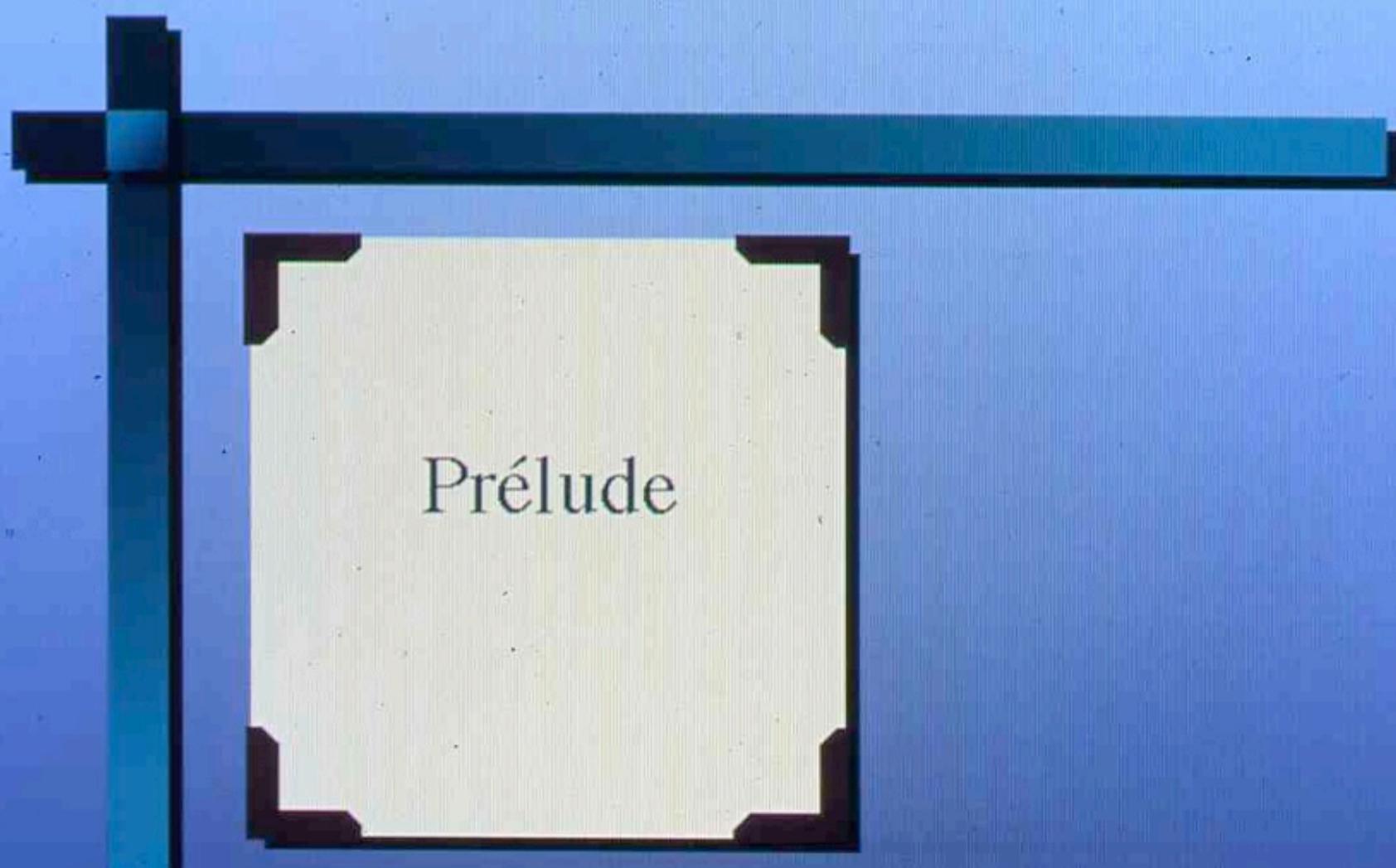


La beauté mathématique des arbres



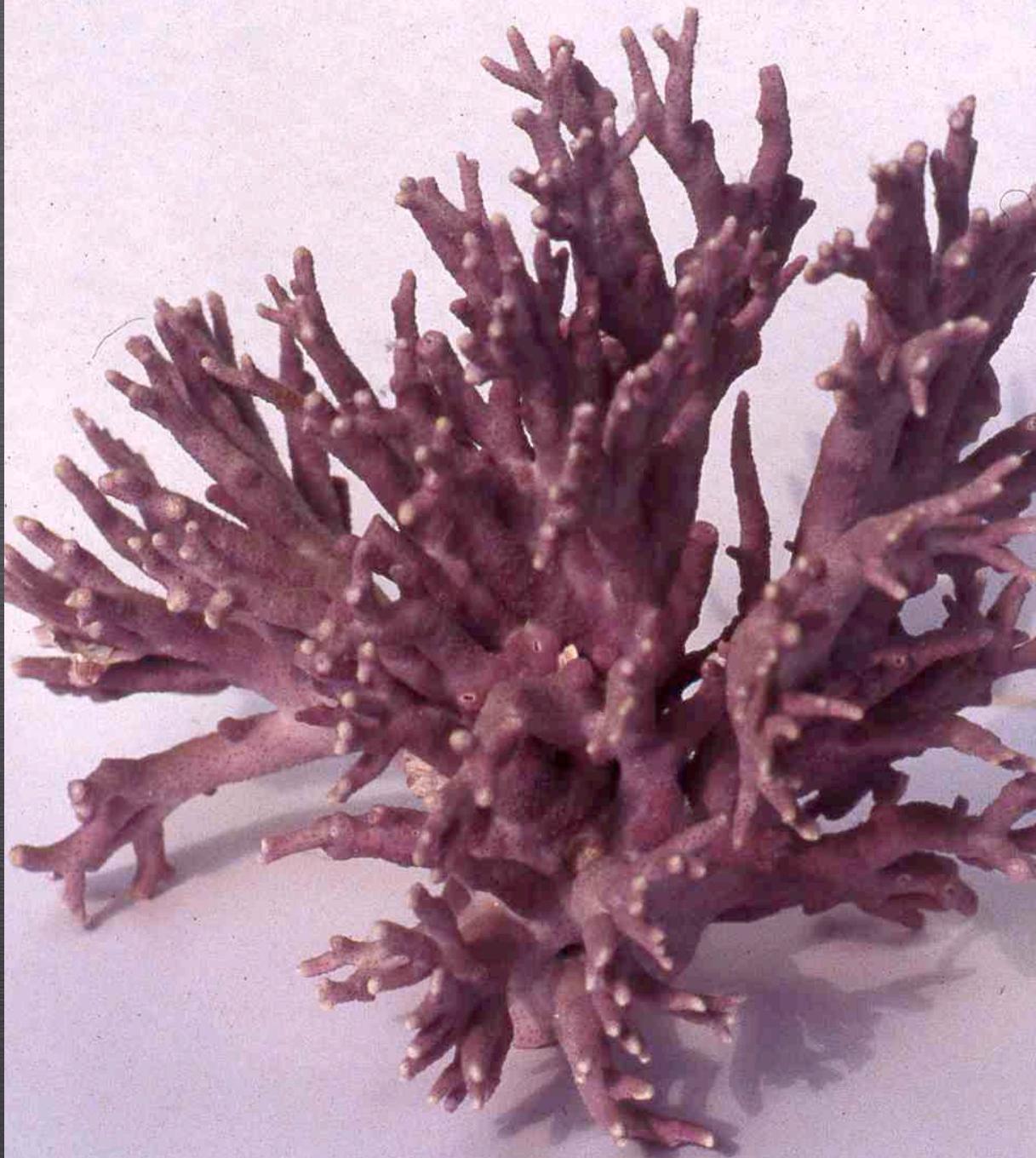


Prélude









comment mesurer
la forme
d'un arbre ?

Quantifier la forme d'un arbre binaire ...

arbre touffu

effilé

broussailleux

épineux

bien équilibré







- "On growth and form"
d'Arcy Thomson 1952
- "Patterns in Nature"
Peter S. Stevens 1974
- "The fractals geometry of Nature"
Mandelbrot (1975,1977) 1982





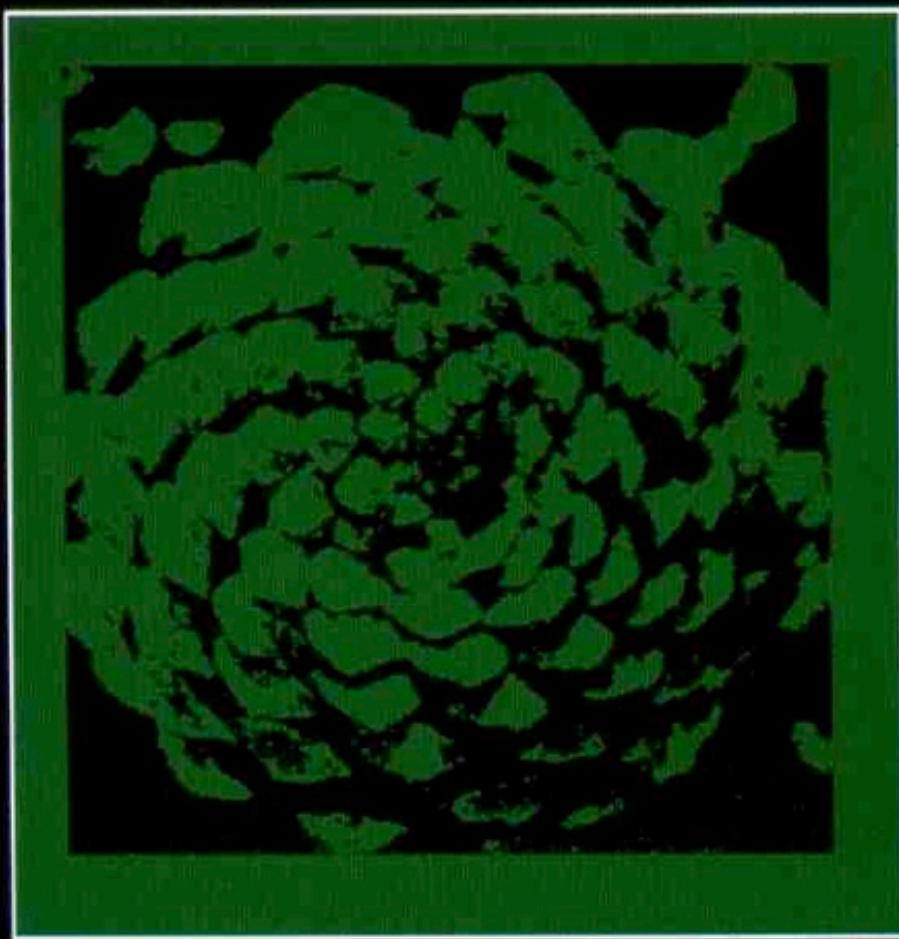
1

Introduction





Derrière les algorithmes
générant ces images d'arbres
il y a de profondes
et très belles
mathématiques combinatoires





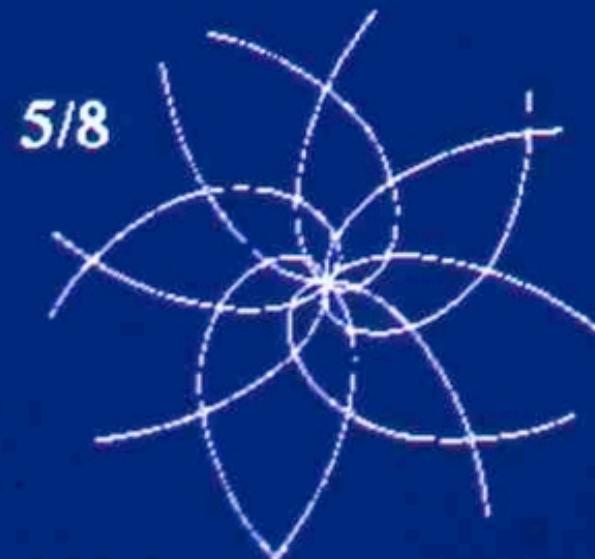
$1/2$



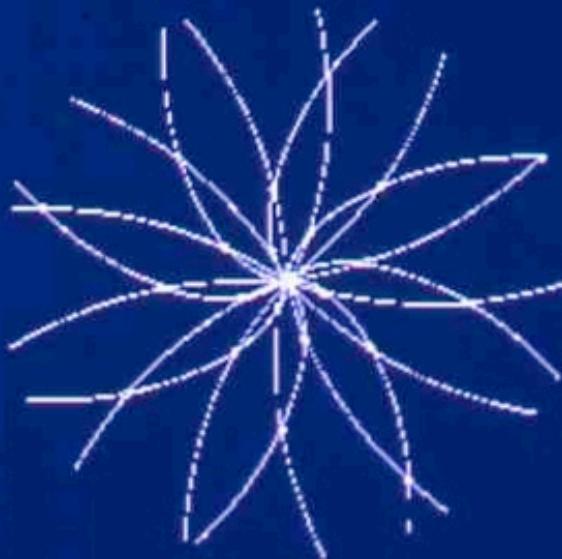
$2/3$



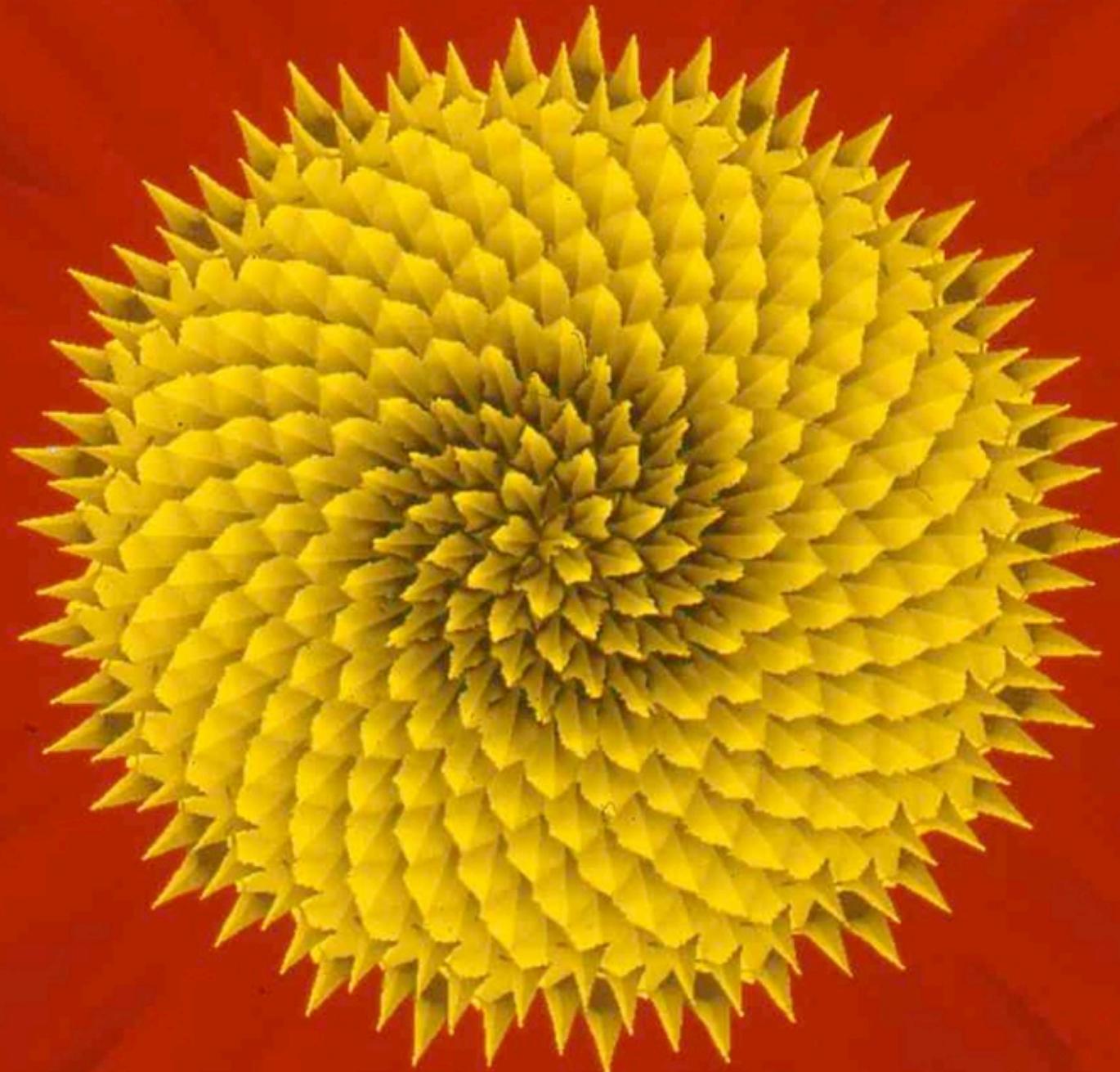
$3/5$



$5/8$



$8/13$



arbre botanique
structure ramifiée



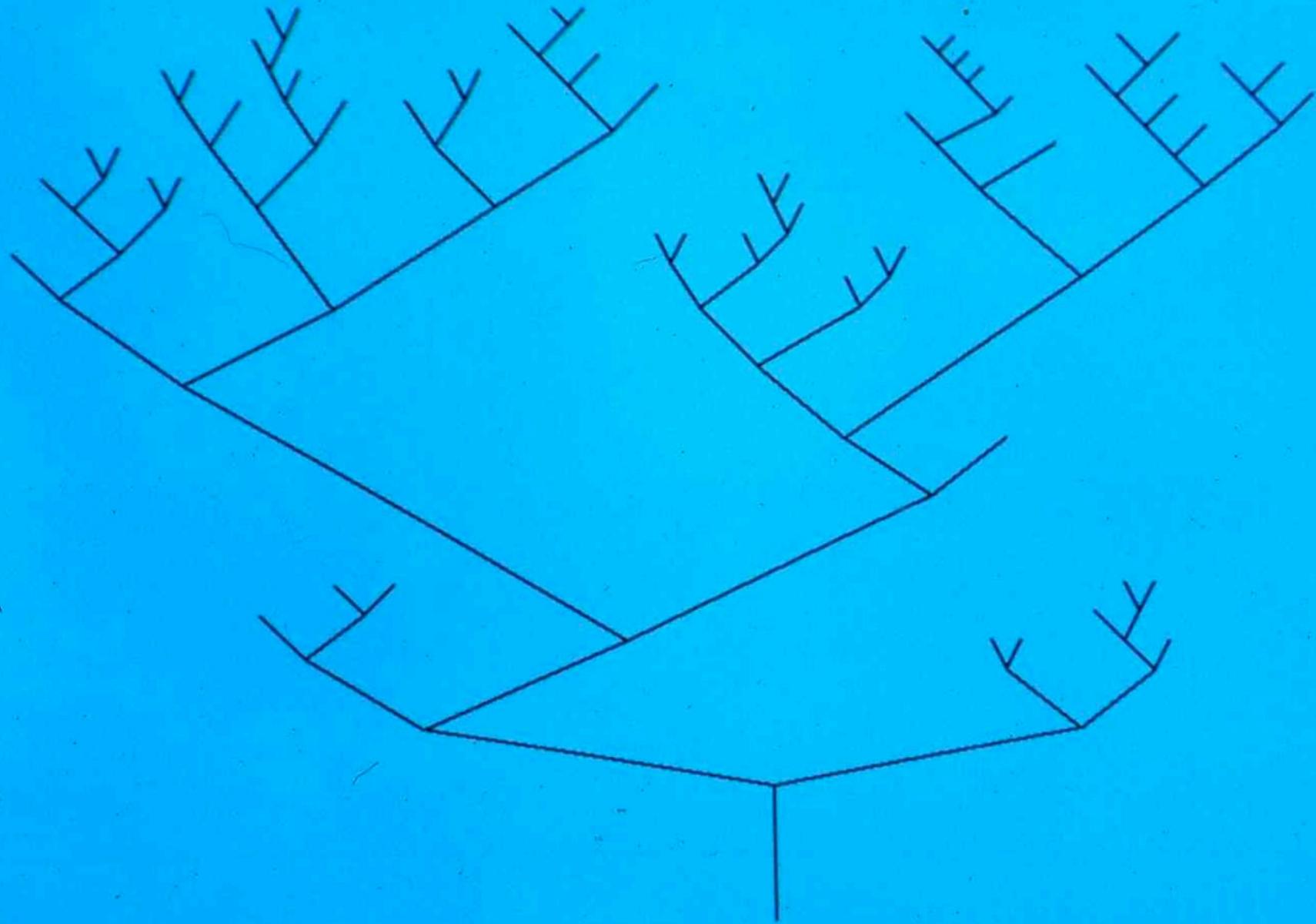
**arbre binaire
combinatoire
(topologique)**



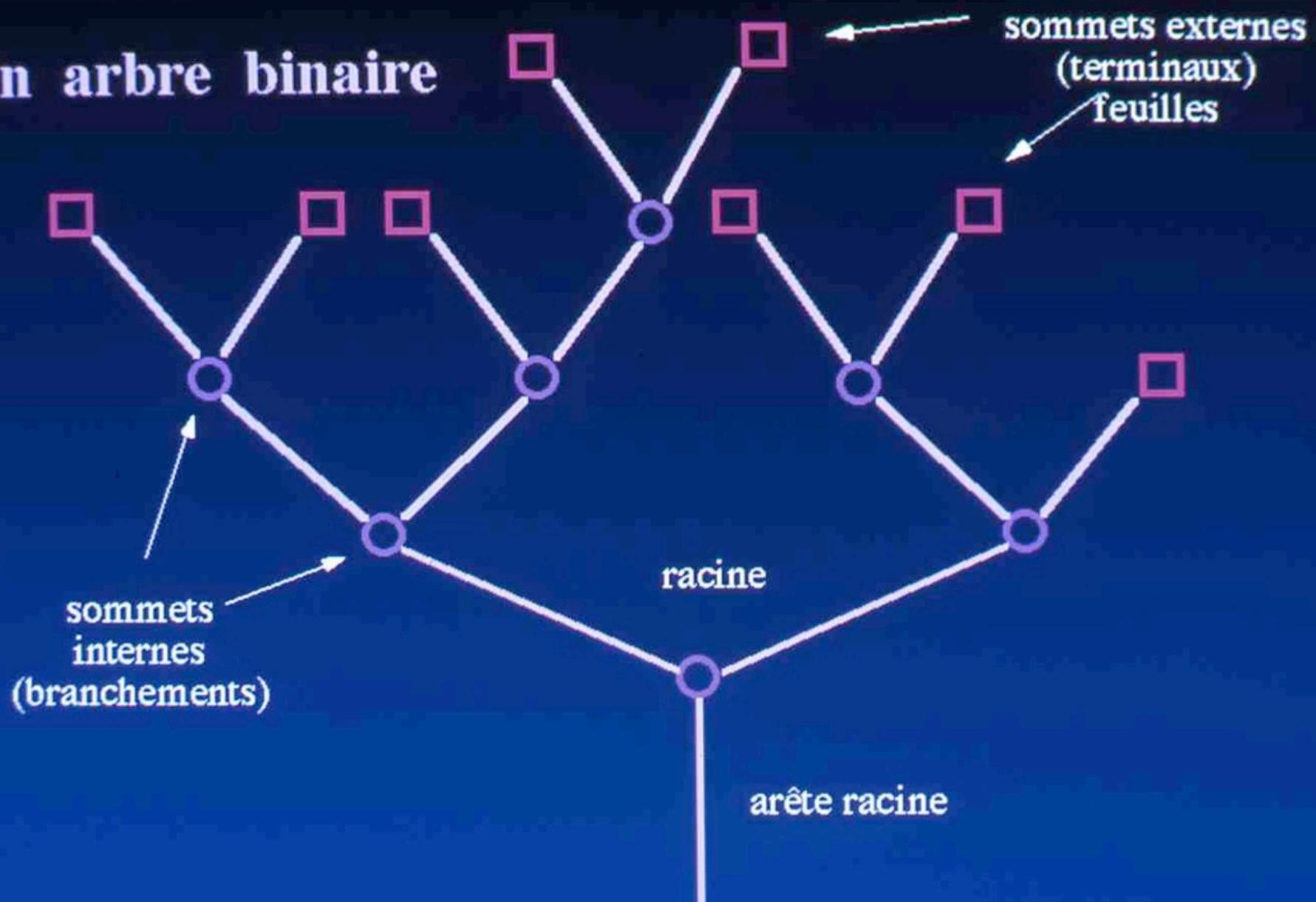








Un arbre binaire









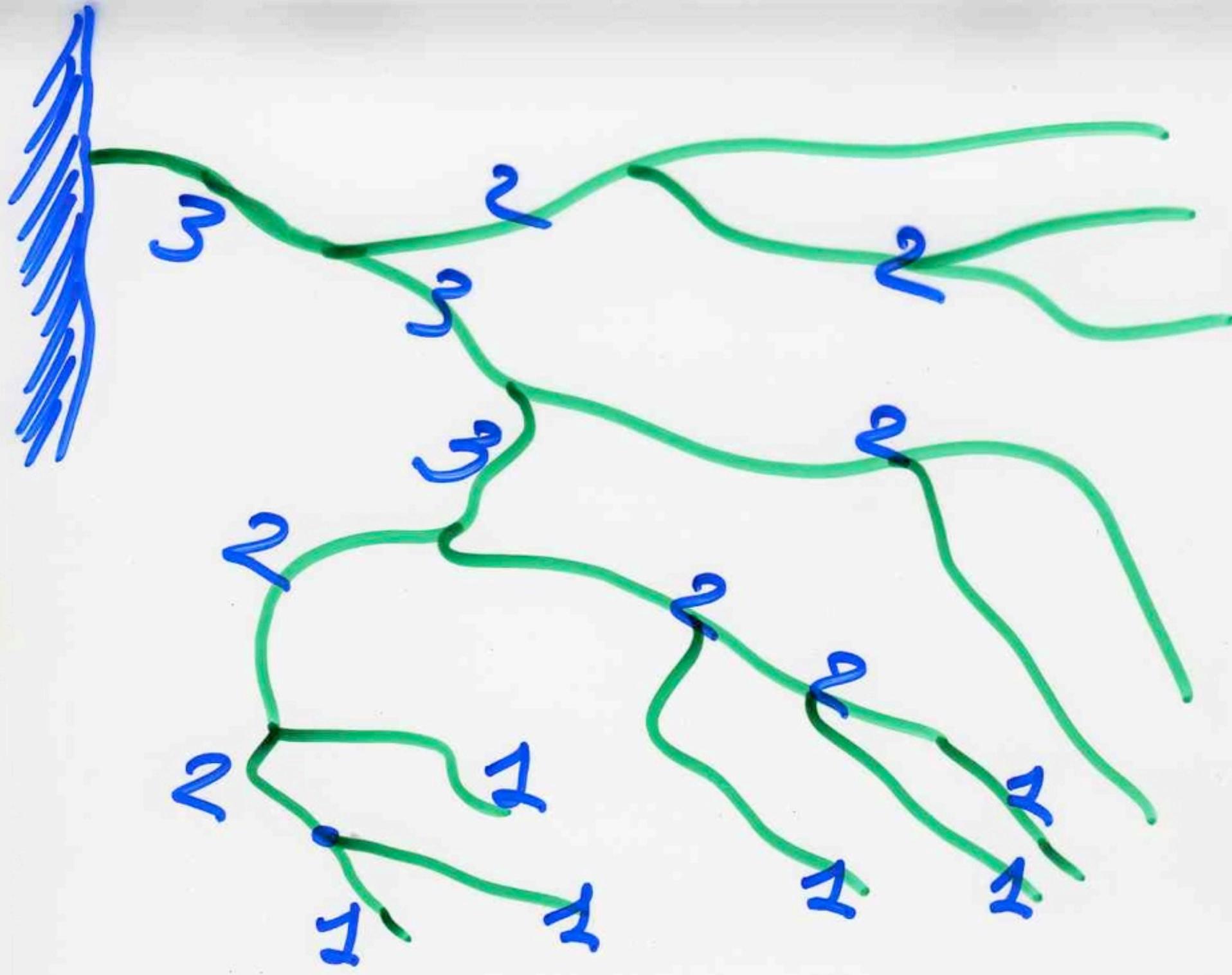
2

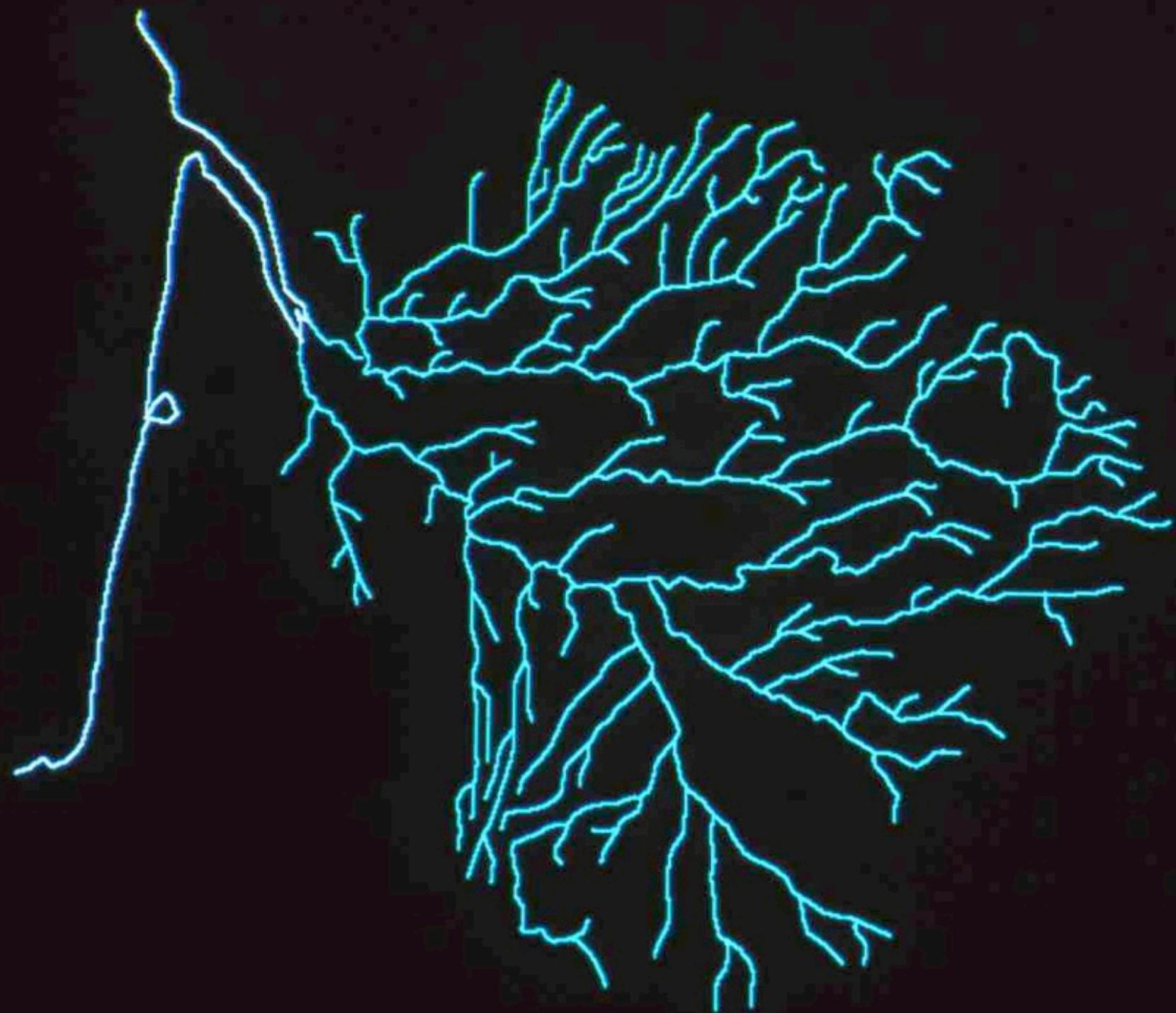
Hydrogéologie

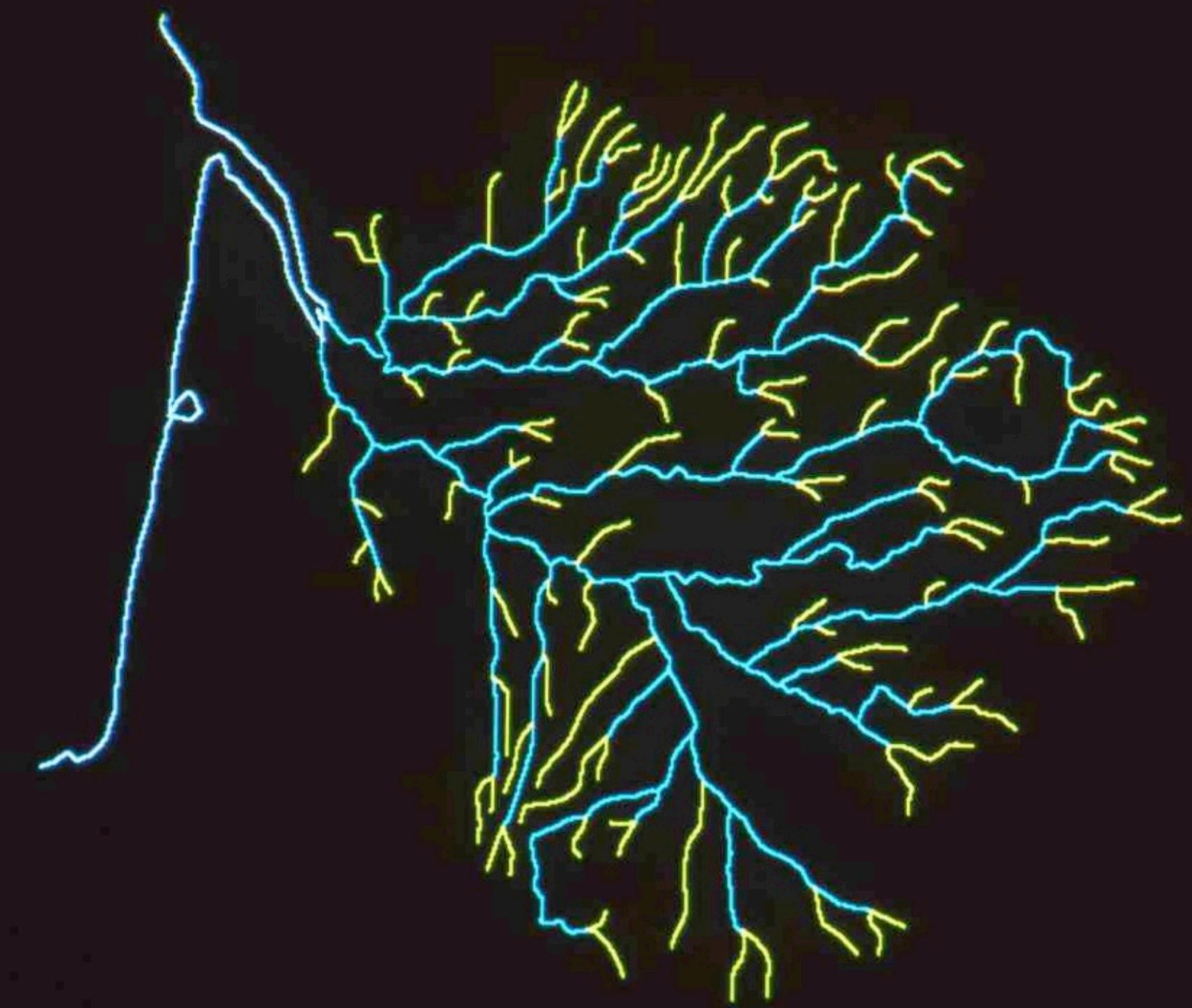
Hydrogéologie

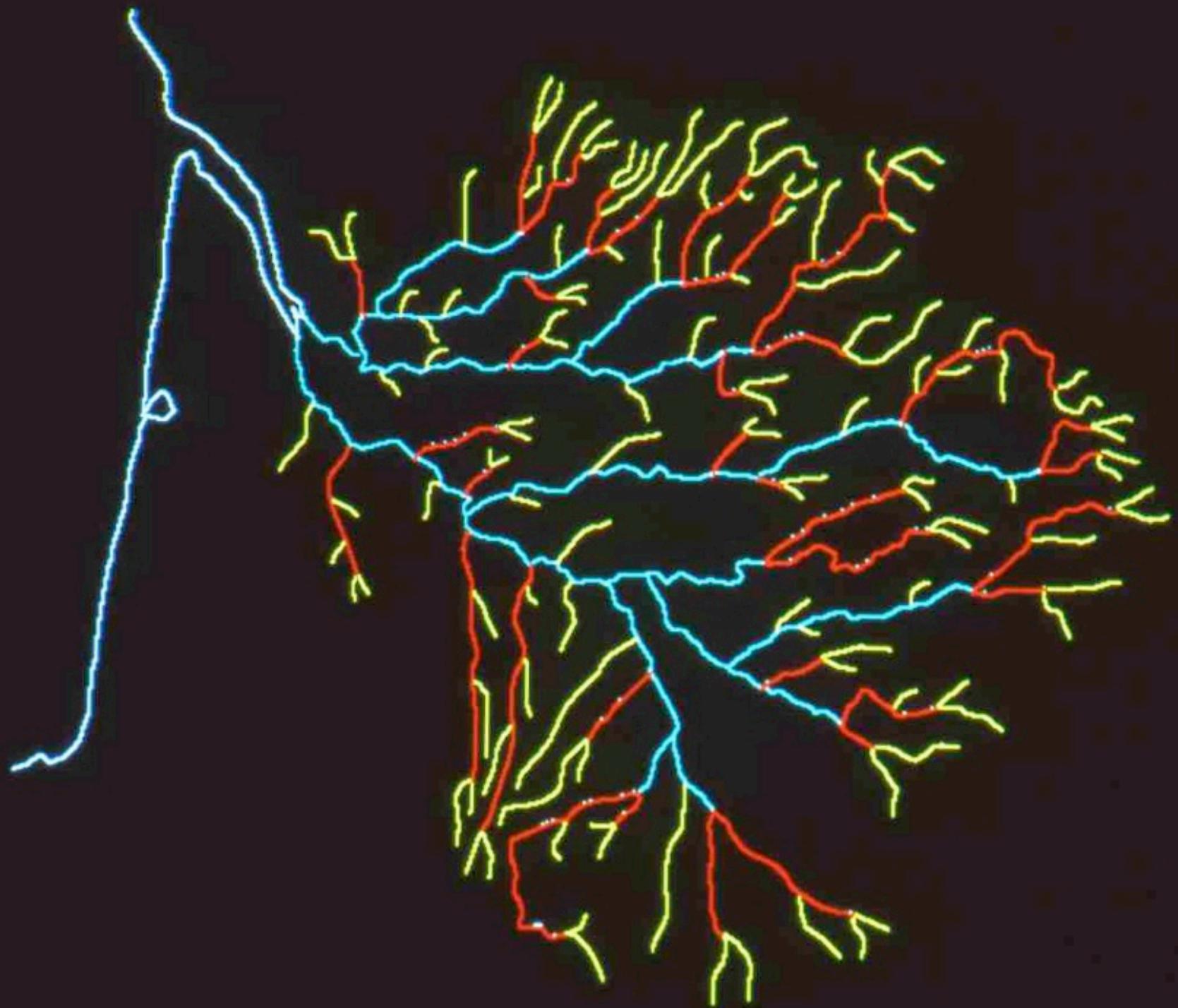
Etude morphologique
des
bassins fluviaux

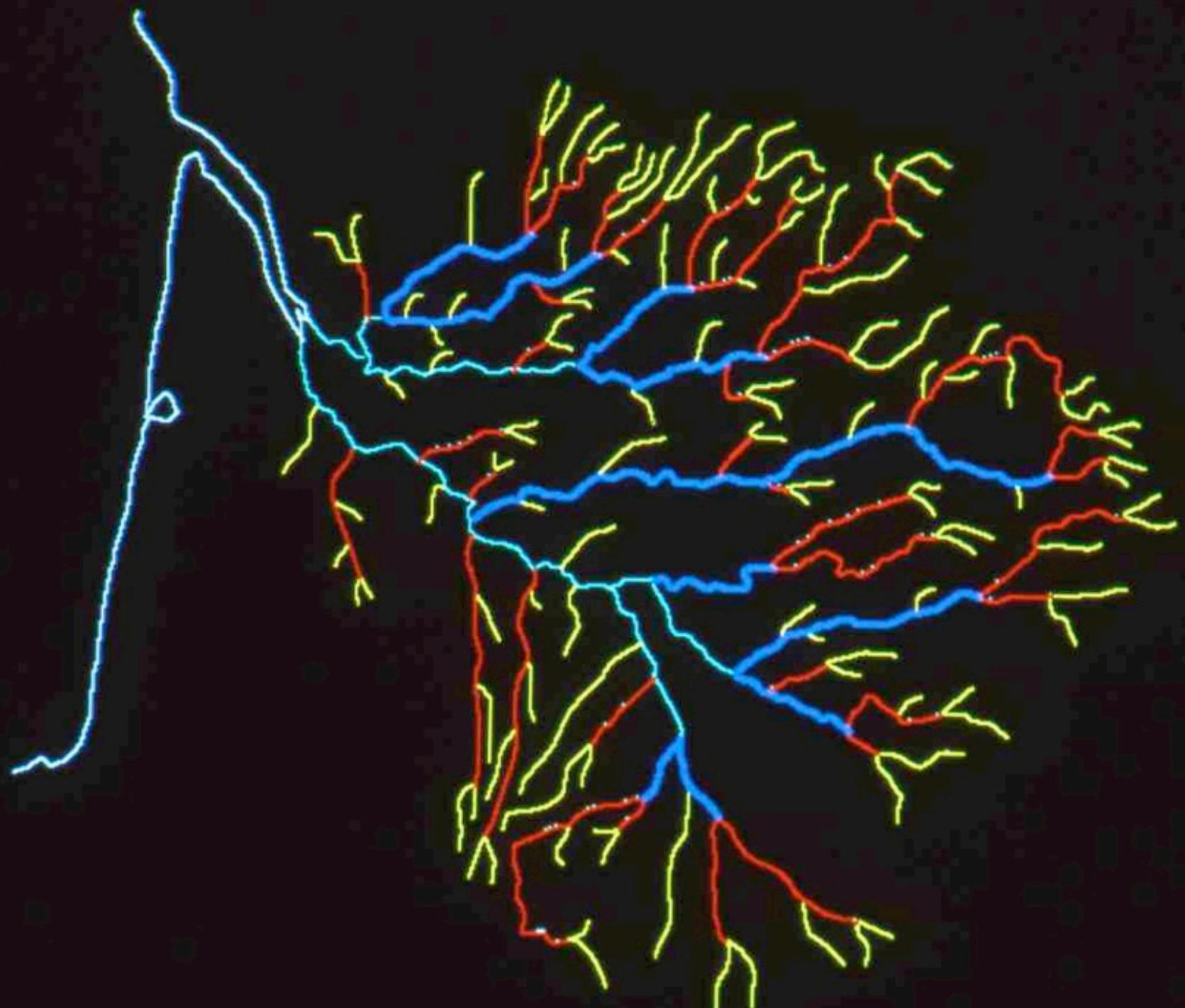
Horton 1945
Strahler 1952

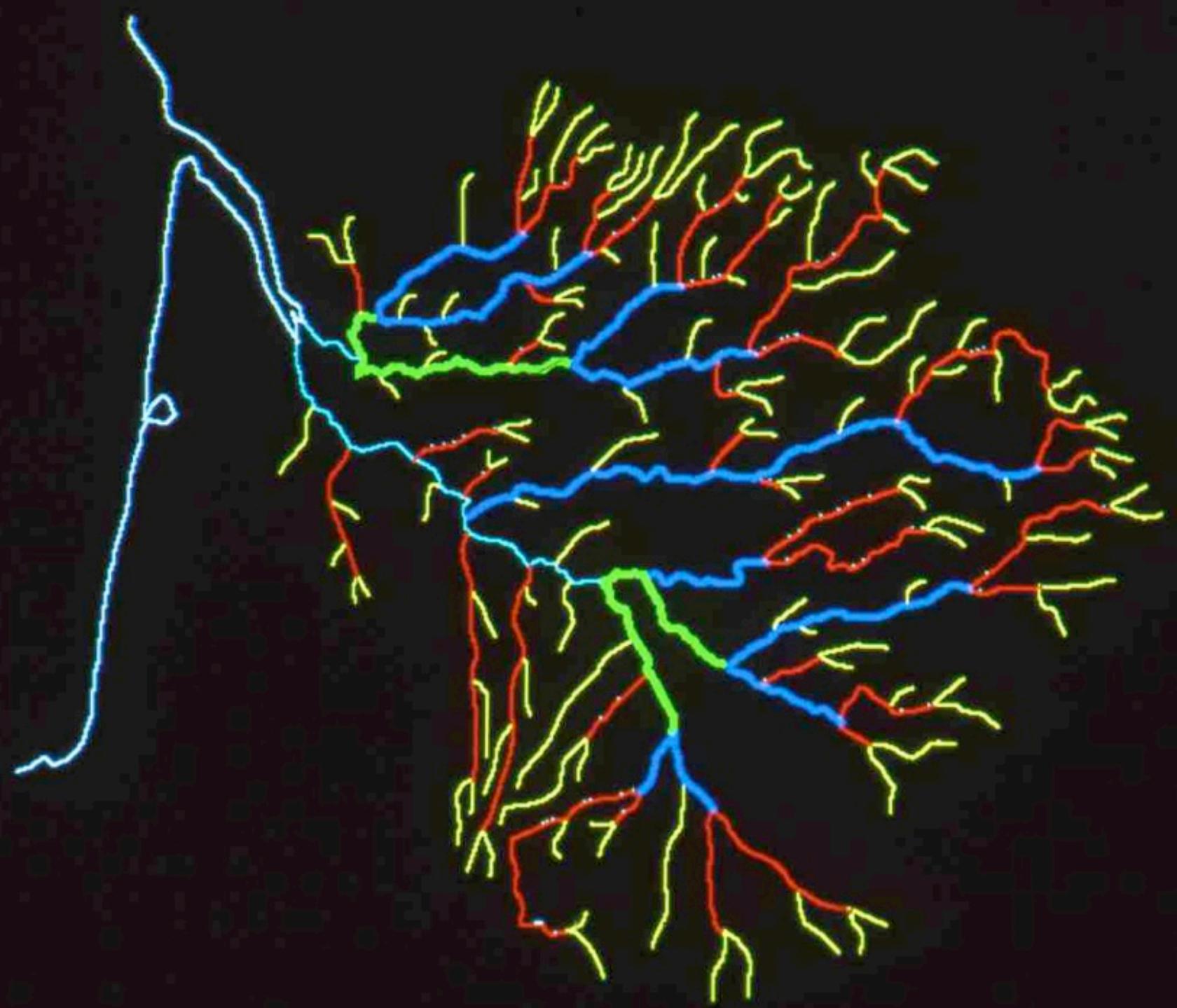


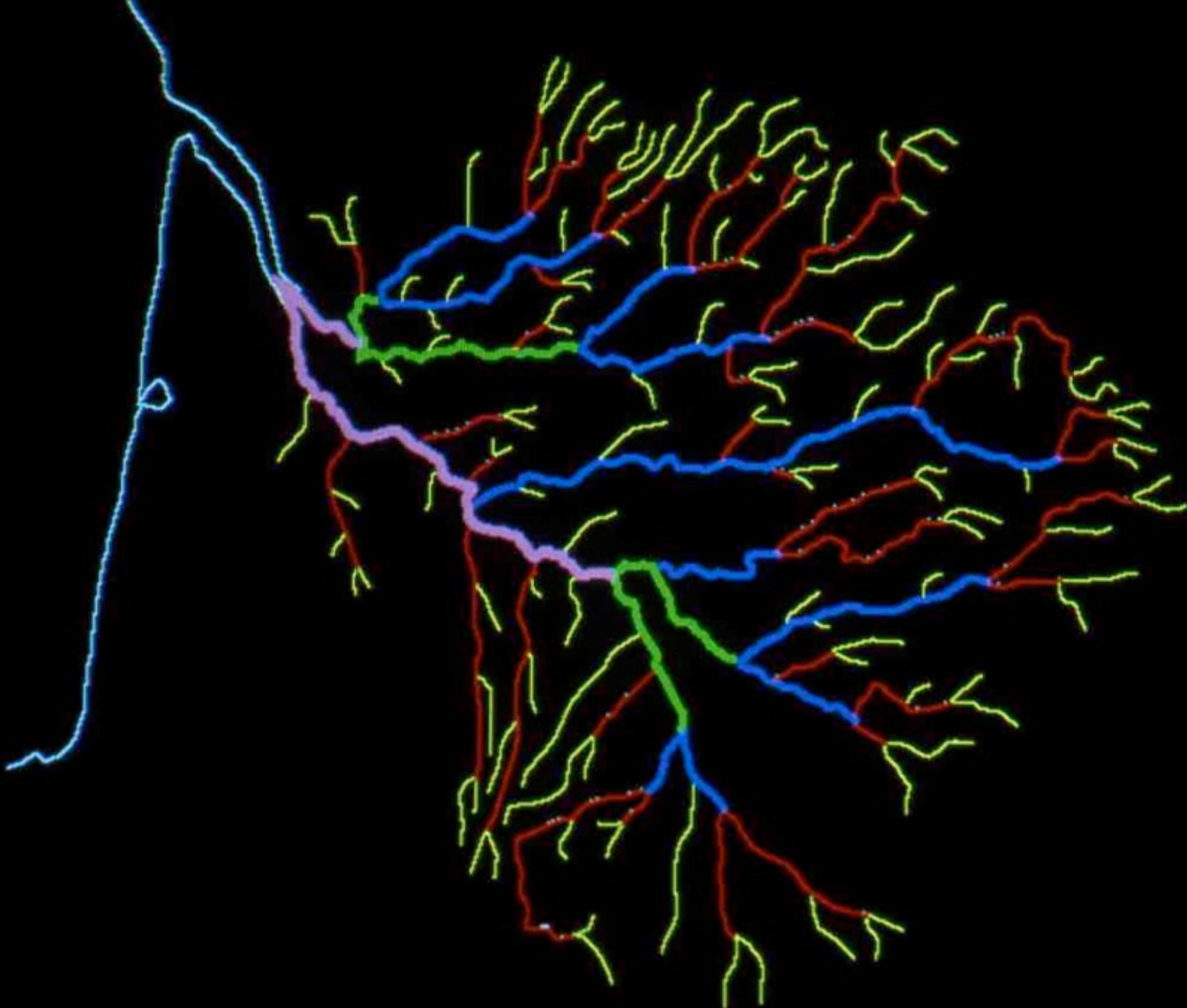


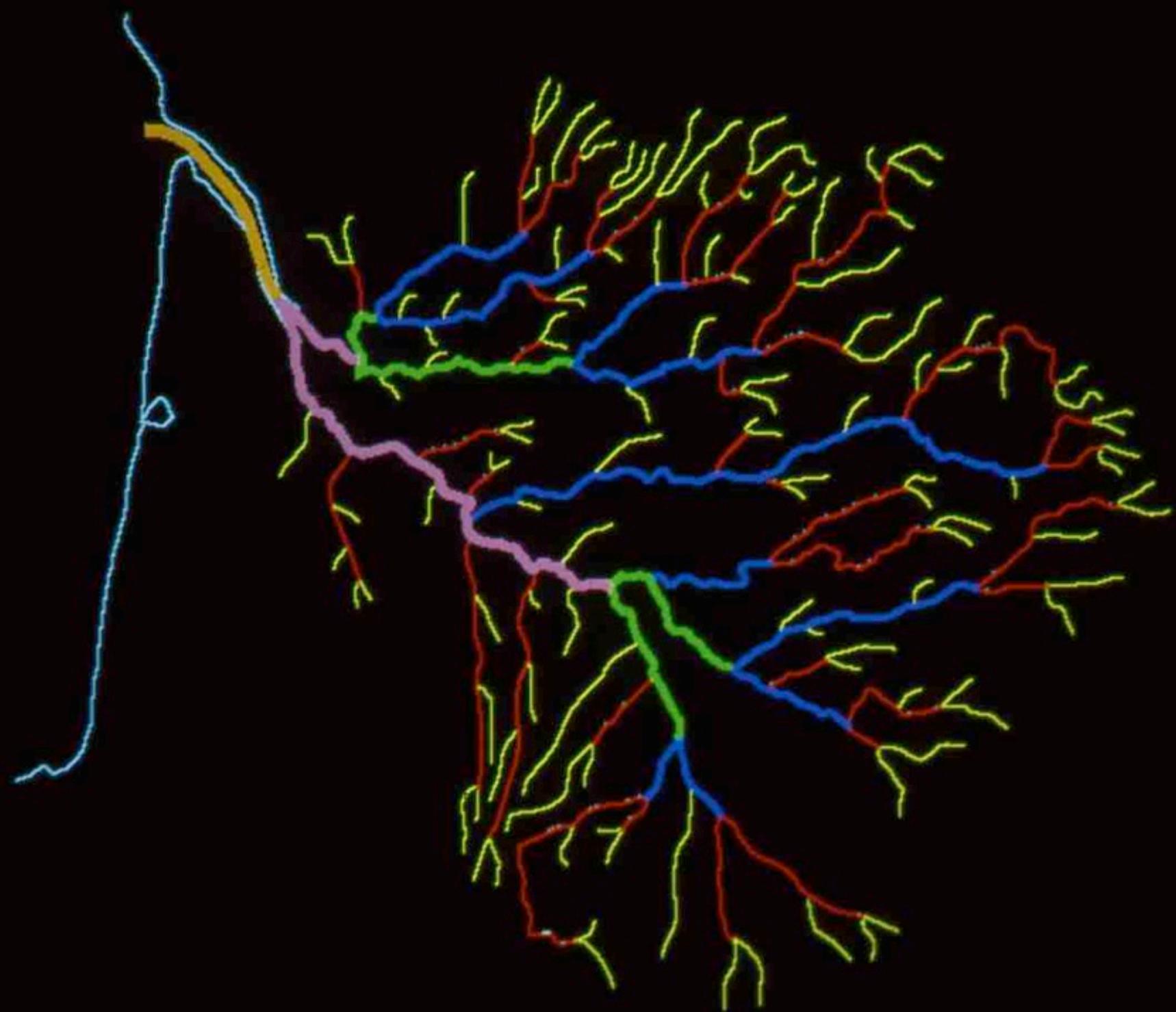












Segment of order k

k

k

$k-1$

$k-1$

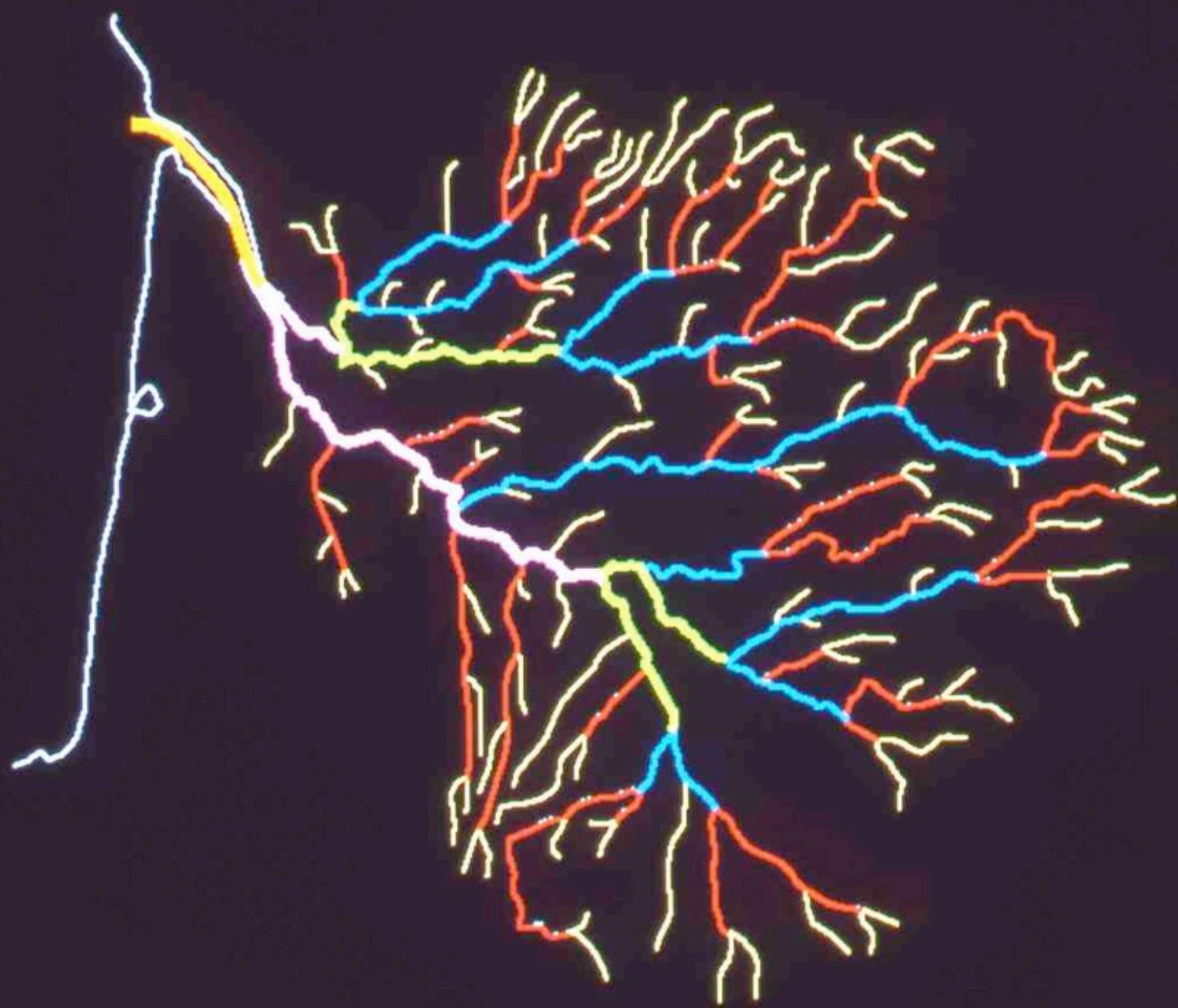
$K > k$

bifurcation ratio

$$3 < \beta_k = \beta < 5$$

$$\beta_k = \frac{b_k}{b_{k+1}}$$

b_k = number of segments
of order k



Corrélation

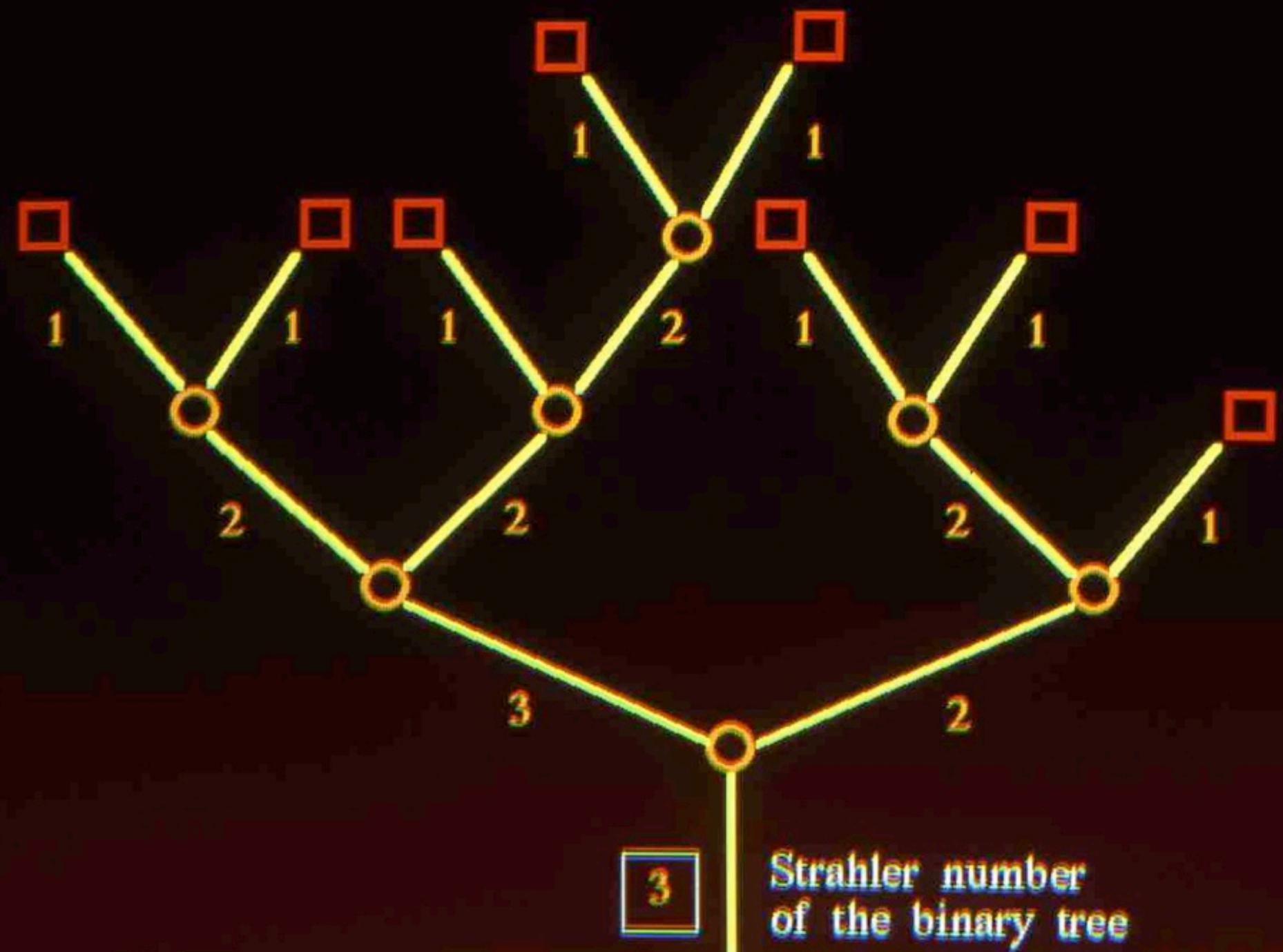
**analyse de Horton-Strahler
&
structure du sous-sol**

**Prud'homme, Naudin, Vigneaux
1970, 1980**

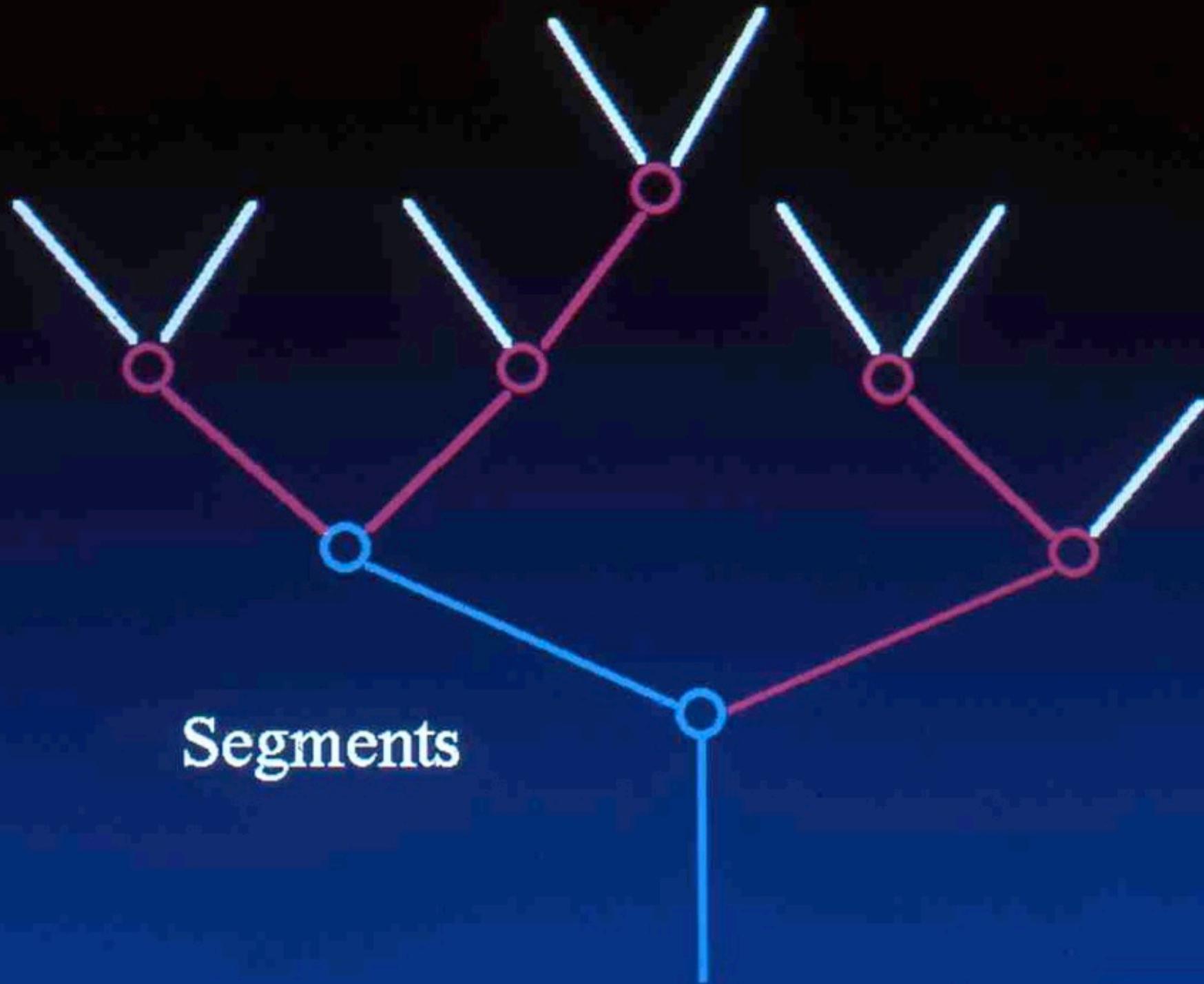


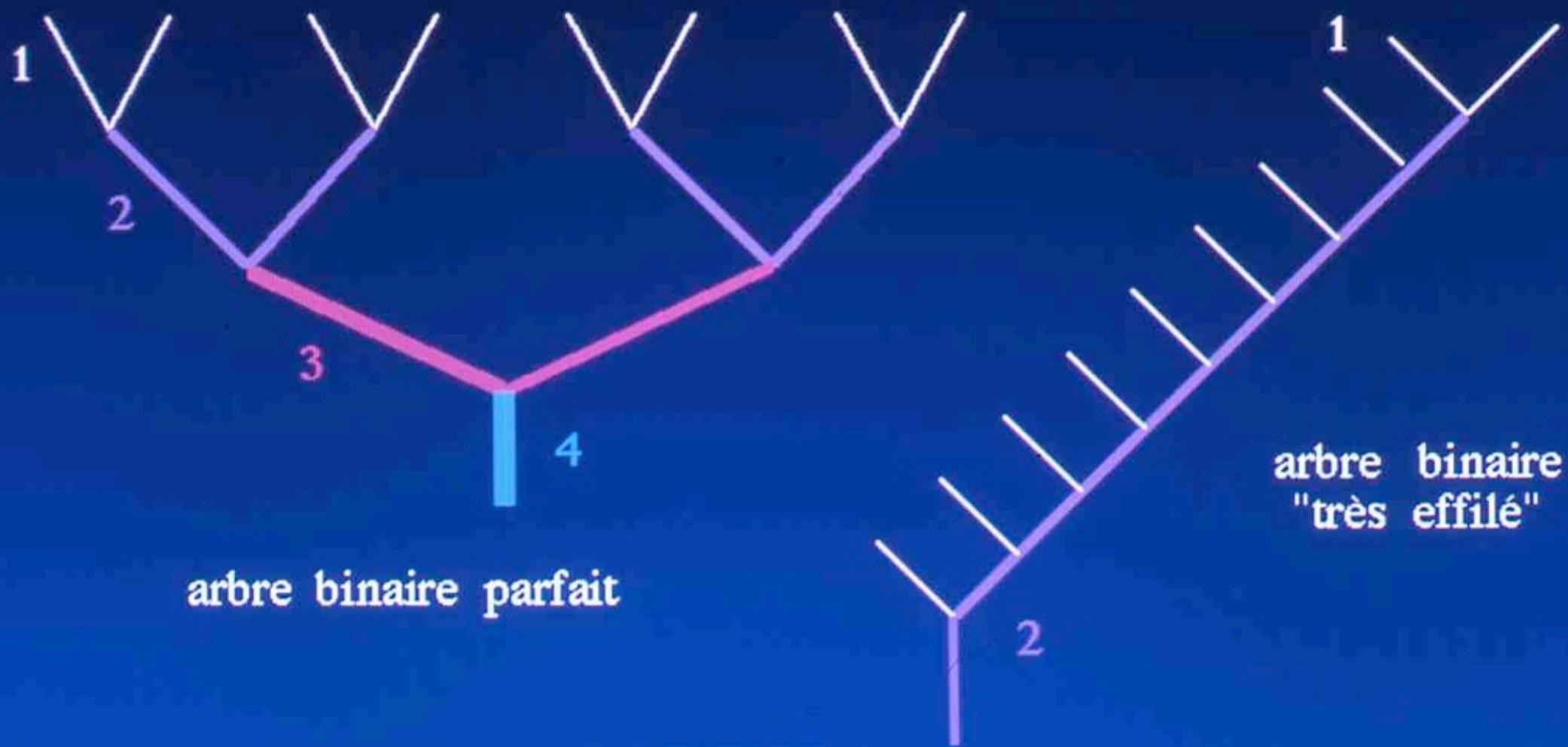
3

Analyse de
Horton-Strahler
pour les arbres
binaires
aléatoires



Segments





bifurcation ratio

s_k^n = mean numbers of segments of order k

(among all binary trees with n internal vertices)

$$\frac{s_k^n}{s_{k+1}^n} = 4 - \frac{4^k}{2n} + O\left(\frac{1}{n^2}\right)$$

$$\beta_k \rightarrow \beta = 4$$

Moon
1980

length segments ratio

$L_k^n = \text{mean of total length of segments of order } k$

(among all binary trees with n internal vertices)

$$\lambda_k \rightarrow \lambda = \beta/2 = 2$$

$$L_k^n = \frac{n}{2^k} + \frac{1}{3} (2^k - 2^{-k}) + O\left(\frac{1}{n}\right)$$

Meir
Moon
1980



4

Problèmes
d'optimisation
en
informatique
théorique

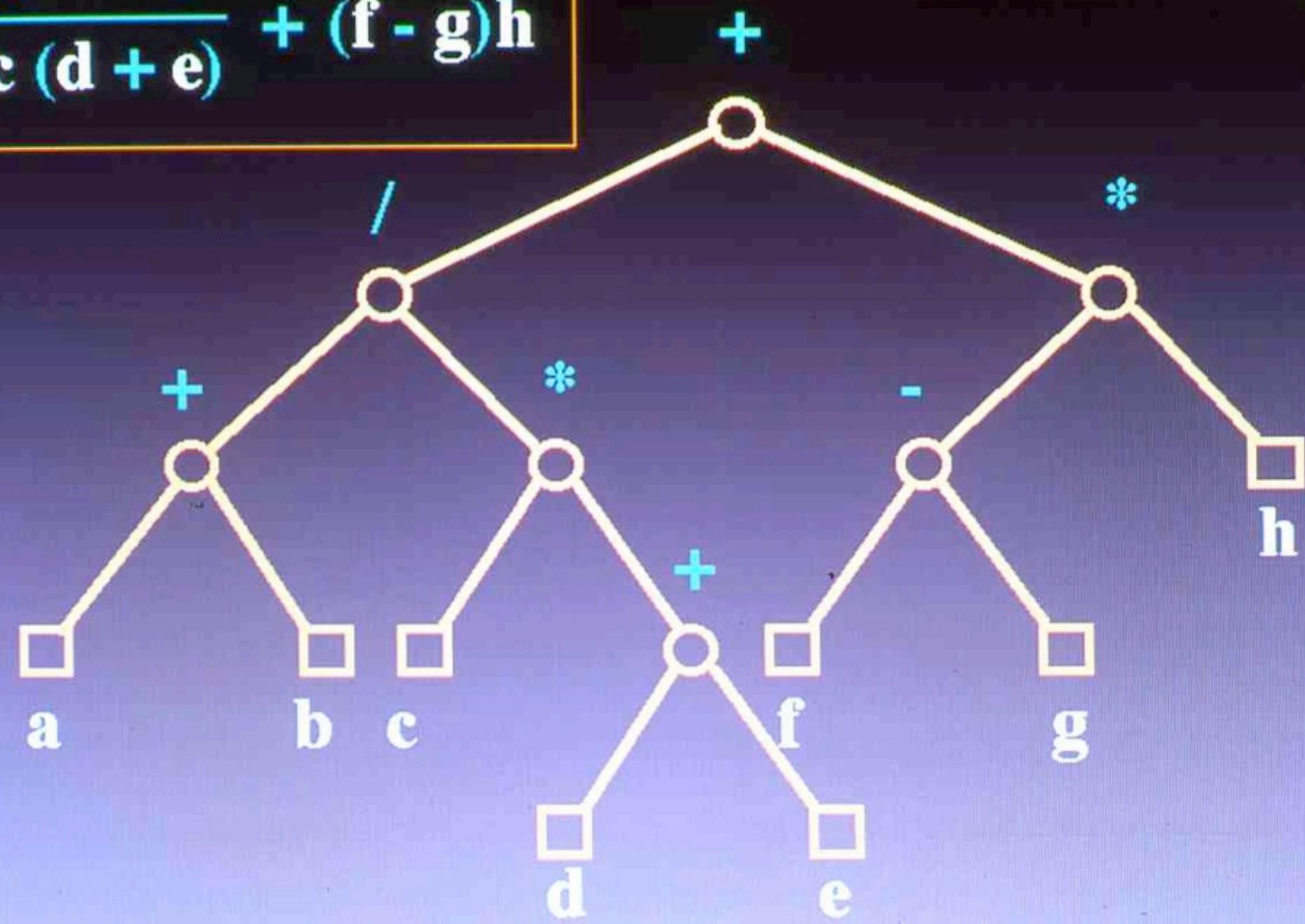
Informatique théorique

Nombre minimum de registres nécessaires pour calculer une expression arithmétique

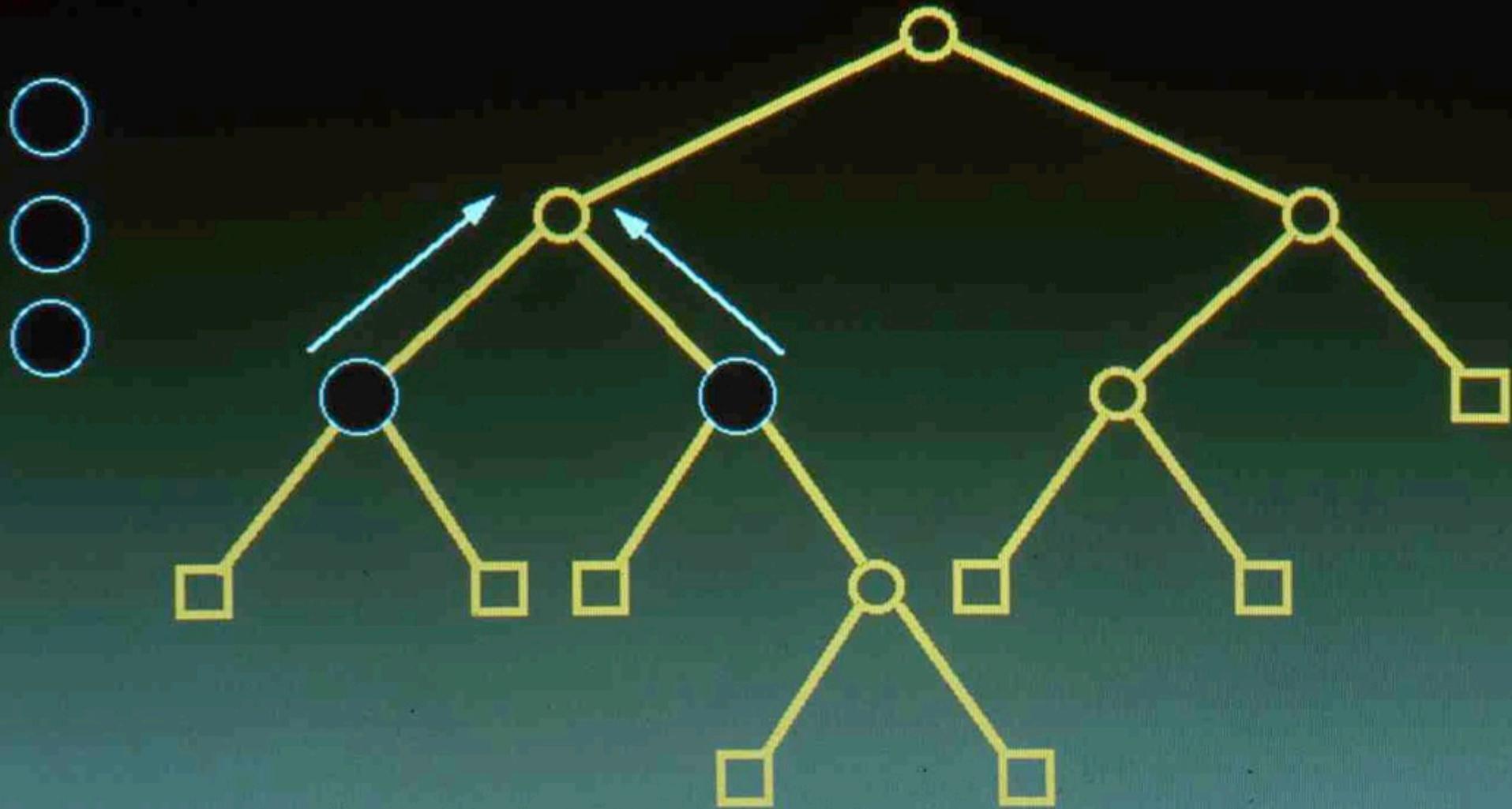
Flajolet, Raoult, Vuillemin
Kemp 1979

$$\frac{(a + b)}{c(d + e)} + (f - g)h$$

Expression arithmétique

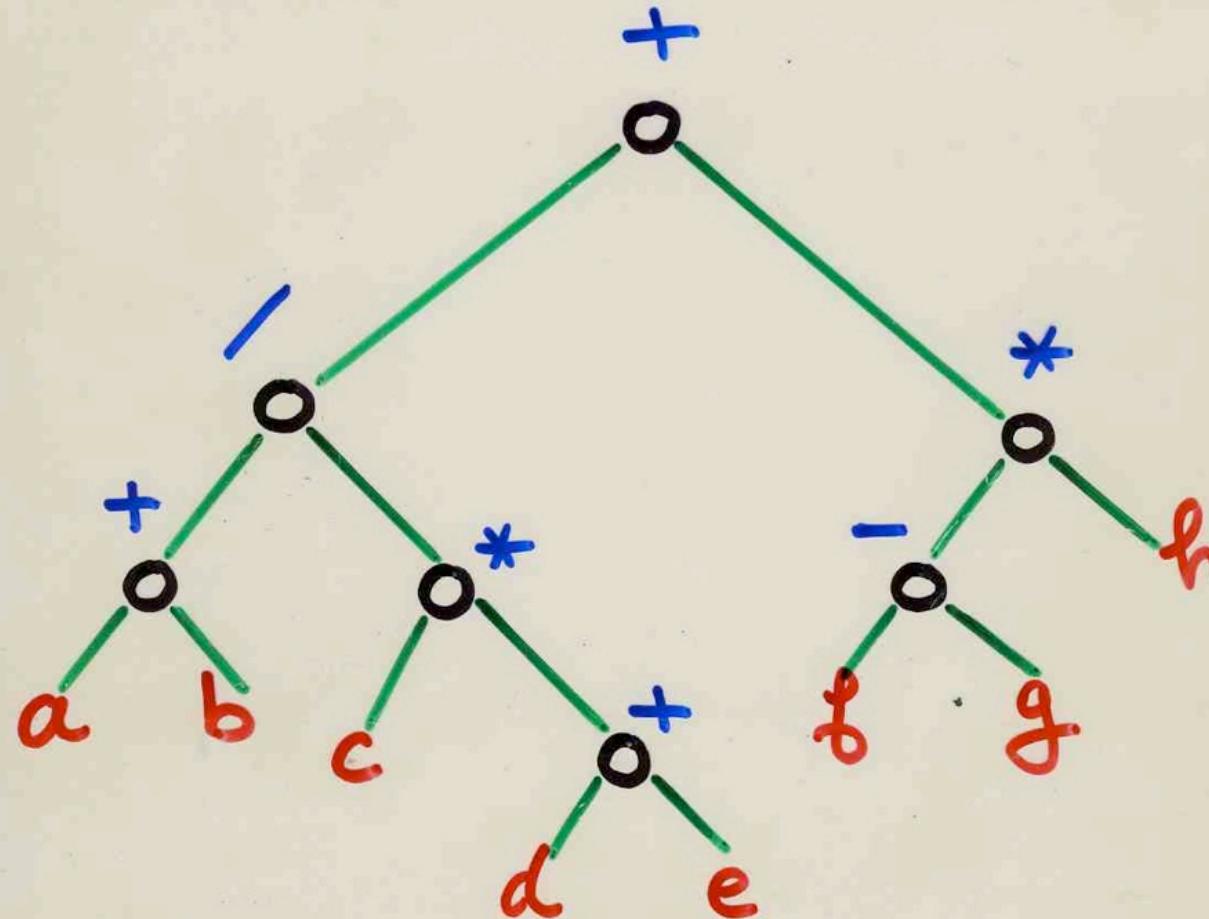


Pebbles problem

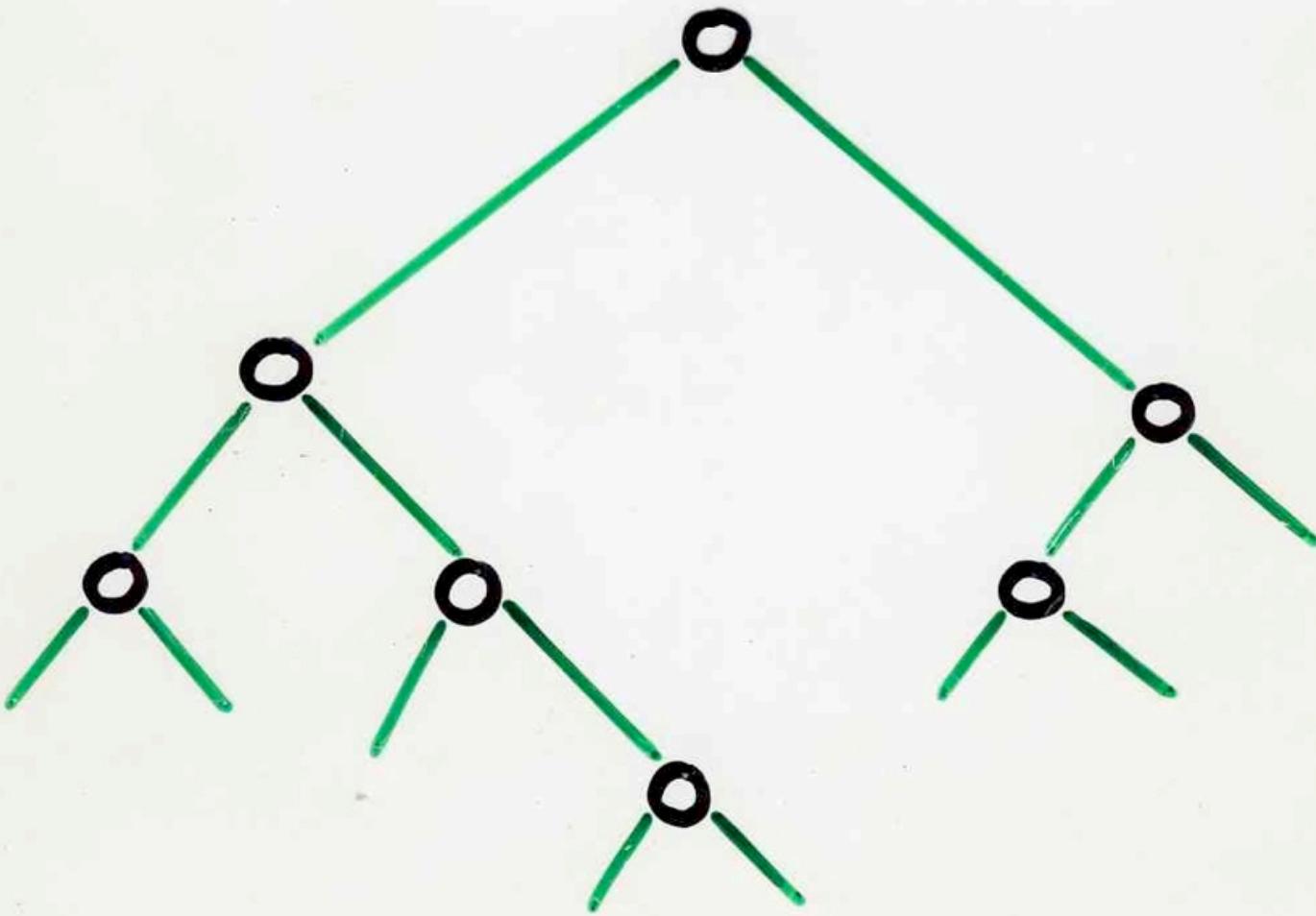


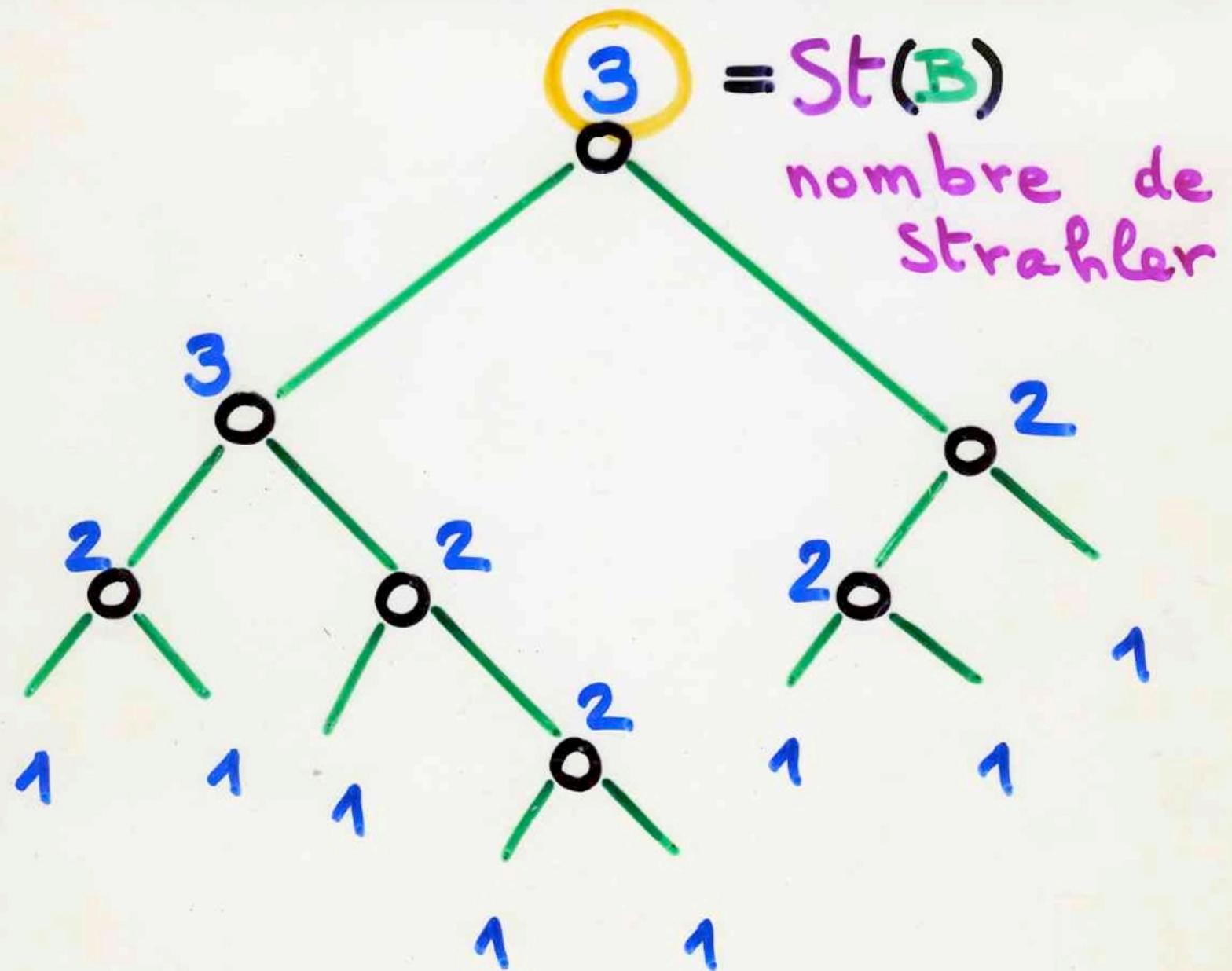
minimum number
of registers
needed to compute an
arithmetical expression

$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c(d+e)} + (f-g)h$$





Nombre de Strahler moyen d'un arbre binaire ayant n sommets

$$A = \log n + f(\log n) + O(1)$$

f fonction⁴ périodique

(Flajolet, Raoult, Vuillemin, Kemp)

$T(n)$ nombre total de 1 parmi
tous les nombres $1, 2, \dots, n-1$
écrits en binaires

Fonction zeta de Riemann

$$\zeta(s) = \sum_{n>0} \frac{1}{n^s}$$



5

combinatoire
des
nombres de
Strahler

$C_n =$ nombre
d'arbres binaires
ayant n sommets internes
(et donc $n+1$ feuilles)

nombre de Catalan

recurrence

$$c_{n+1} = \sum_{i+j=n} c_i c_j$$

$$c_0 = 1$$

$c_0 \ c_1 \ c_2 \ c_3 \ c_4 \ c_5$
1, 1, 2, 5, 14, 42, ...

$$c_6 = c_0 c_5 + c_1 c_4 + c_2 c_3 + c_3 c_2 + c_4 c_1 + c_5 c_0$$

132 $1 \times 42 + 1 \times 14 + 2 \times 5 + 5 \times 2 + 14 \times 1 + 42 \times 1$

$$c_n = \frac{1}{n+1} \binom{2n}{n}$$

$$= \frac{(2n)!}{(n+1)! n!}$$

$$n! = 1 \times 2 \times \dots \times n$$

C_{2n}^n

$$n! = 1 \times 2 \times \dots \times n$$

$$C_4 = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}{1 \times 2 \times 3 \times 4 \quad 1 \times 2 \times 3 \times 4 \times 5}$$

combinatoire
énumérative
classique

Note sur une Équation aux différences finies;

PAR E. CATALAN.

M. Lamé a démontré que l'équation

$$P_{n+1} = P_n + P_{n-1}P_3 + P_{n-2}P_4 + \dots + P_4P_{n-3} + P_3P_{n-2} + P_n, \quad (1)$$

se ramène à l'équation linéaire très simple,

$$P_{n+1} = \frac{4n-6}{n} P_n. \quad (2)$$

Admettant donc la concordance de ces deux formules, je vais chercher à en déduire quelques conséquences.

I.

L'intégrale de l'équation (2) est

$$P_{n+1} = \frac{6}{3} \cdot \frac{10}{4} \cdot \frac{14}{5} \cdots \frac{4n-6}{n} P_1;$$

et comme, dans la question de géométrie qui conduit à ces deux équations, on a $P_1 = 1$, nous prendrons simplement

$$P_{n+1} = \frac{2 \cdot 6 \cdot 10 \cdot 14 \cdots (4n-6)}{2 \cdot 3 \cdot 4 \cdot 5 \cdots n}. \quad (3)$$

Le numérateur

$$\begin{aligned} 2 \cdot 6 \cdot 10 \cdot 14 \cdots (4n-6) &= 2^{n-1} \cdot 1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-3) \\ &= \frac{2^{n-1} \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdots (2n-2)}{2 \cdot 4 \cdot 6 \cdot 8 \cdots (2n-2)} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdots (2n-2)}{1 \cdot 2 \cdot 3 \cdots (n-1)}. \end{aligned}$$

Donc

$$P_{n+1} = \frac{n(n+1)(n+2)\cdots(2n-2)}{2 \cdot 3 \cdot 4 \cdots n}. \quad (4)$$

Si l'on désigne généralement par $C_{m,p}$ le nombre des combinaisons de m lettres, prises p à p ; et si l'on change n en $n+1$, on aura

$$P_{n+1} = \frac{1}{n+1} C_{2n,n}, \quad (5)$$

ou bien

$$P_{n+1} = C_{2n,n} - C_{2n,n-1}. \quad (6)$$

II.

Les équations (1) et (5) donnent ce théorème sur les combinaisons :

$$\left. \begin{aligned} \frac{1}{n+1} C_{2n,n} &= \frac{1}{n} C_{2n-2,n-1} + \frac{1}{n-1} C_{2n-4,n-2} \times \frac{1}{2} C_{2,2} \\ &\quad + \frac{1}{n-2} C_{2n-6,n-3} \times \frac{1}{3} C_{4,2} + \dots + \frac{1}{n} C_{2n-2,n-1}. \end{aligned} \right\} \quad (7)$$

III.

On sait que le $(n+1)^{\text{e}}$ nombre figuré de l'ordre $n+1$, a pour expression, $C_{2n,n}$: si donc, dans la table des nombres figurés, on prend ceux qui occupent la diagonale; savoir :

$$1, 2, 6, 20, 70, 252, 924\dots;$$

qu'on les divise respectivement par

$$1, 2, 3, 4, 5, 6, 7\dots;$$

on obtiendra une nouvelle suite de nombres,

$$1, 1, 2, 5, 14, 42, 132\dots, \quad (\Lambda)$$

lesquels jouiront de cette propriété :

Un terme quelconque de la suite (Λ) est égal à la somme des produits que l'on obtient en écrivant au-dessous d'elle-même, et dans un ordre inverse, la série des termes précédents, et en multipliant les termes correspondants des deux séries.

Par exemple,

$$132 = 1 \cdot 42 + 1 \cdot 14 + 2 \cdot 5 + 5 \cdot 2 + 14 \cdot 1 + 42 \cdot 1.$$

addition +

A

11

1. 2. 1.

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1
1 1
1 2 2
1 3 5 5
1 4 9 14 14
1 5 14 28 42 42

.....

①

addition

+

1

1

1

2 - 1

1

3 3 1

1

4 6 - 4 1

1

5 10 10 5 1

1

6 15 20 - 15 6 1

1

7 21 35 35 21 7 1

1

8 28 56 70 - 56 28 8 1

 1

$$\begin{array}{r} 1 \quad 1 \\ 1 \quad \underline{2} \\ 2 \end{array}$$

$$\begin{array}{r} 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad \underline{3} \quad 3 \quad 1 \end{array}$$

$$\begin{array}{r} 1 \quad 4 \quad \underline{6} \quad 4 \quad 1 \\ 1 \quad 4 \quad \underline{6} \quad 4 \quad 1 \end{array}$$

$$\begin{array}{r} 1 \quad 5 \quad 30 \quad 10 \quad 5 \quad 1 \\ 1 \quad 5 \quad \underline{30} \quad 10 \quad 5 \quad 1 \end{array}$$

$$\begin{array}{r} 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\ 1 \quad 6 \quad 15 \quad \underline{20} \quad 15 \quad 6 \quad 1 \end{array}$$

$$\begin{array}{r} 1 \quad 7 \quad 21 \quad 45 \quad 35 \quad 21 \quad 7 \quad 1 \\ 1 \quad 7 \quad 21 \quad \underline{45} \quad 35 \quad 21 \quad 7 \quad 1 \end{array}$$

$$\begin{array}{r} 1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \\ 1 \quad 8 \quad 28 \quad 56 \quad \underline{70} \quad 56 \quad 28 \quad 8 \quad 1 \\ 5 \end{array}$$

addition
division



série génératrice
des nombres c_n

série formelle

1 1 2 5 14 42

$$1 + 1t + 2t^2 + 5t^3 + 14t^4 + 42t^5$$

$$1 + 1t + 2t^2 + 5t^3 + 14t^4 + 42t^5$$

+ ...

$$(1+t)^2 = 1 + 2t + t^2$$

$$(1+t)^3 = 1 + 3t + 3t^2 + t^3$$

$$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \dots + t^n$$

+ ...

$$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \dots + t^n + \dots$$

$$(1 + t + t^2 + \dots + t^n + \dots) \times (1 - t)$$

$$\frac{1}{1-(t+t^2)} = ?$$

$$\frac{1}{1-(t+t^2)} = ?$$

$$\begin{aligned} &= 1 + t + 2t^2 + 3t^3 + 5t^4 \\ &\quad + 8t^5 + 13t^6 + 21t^7 \\ &\quad + 34t^8 + 55t^9 + \dots \end{aligned}$$

$$= 1 + t + 2t^2 + 3t^3 + 5t^4 \\ + 8t^5 + 13t^6 + 21t^7 \\ + 34t^8 + 55t^9 + \dots$$

$$F_n = F_{n-1} + F_{n-2}$$

Fibonacci

$$\sum_{i>0} (t + t^2)^i =$$
$$1 + (t + t^2)$$
$$(t^2 + 2t^3 + t^4)$$
$$(t^3 + 3t^4 + 3t^5 + t^6)$$
$$(t^4 + 4t^5 + 6t^6 + \dots)$$
$$+ (t^5 \dots \dots \dots)$$

$$\sum_{i \geq 0} (t + t^2)^i =$$

$$1 + (t + t^2)$$

$$(t^2 + 2t^3 + t^4)$$

$$(t^3 + 3t^4 + 3t^5 + t^6)$$

$$(t^4 + 4t^5 + 6t^6 + \dots)$$

$$+ (t^5 \dots)$$

↓ ↓ ↓ ↓ ↓ ↓

1 2 3 5 8

$$F_{n+1} = F_n + F_{n-1}$$

$$F_0 = F_1 = 1$$

Fibonacci

$$F_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n$$

$$\frac{F_{n+1}}{F_n}$$



ϕ

nombre
d'or

série génératrice

$$f(t) = a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n + \dots$$

$$f(t) = \sum_{n \geq 0} a_n t^n$$

fonction

$f(t)$

développement de Taylor

au voisinage de $t=0$

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \dots + a_n t^n + \dots$$

$$t + t + t + \dots t + \dots$$
$$1 + 1 + 1 + \dots$$

~~$t + t + t + \dots$~~

~~$1 + 1 + 1 + \dots$~~

Arbre

=



Arbre

Arbre

y

= 1 +

t

y

y

$$y = 1 + t(y)^2$$

$$y = 1 + t y^2$$

equation algébrique

$$ay^2 + by + c = 0$$

$$\frac{-b \pm \sqrt{}}{2a}$$

$$y = \frac{1 - (1 - 4t)^{\frac{1}{2}}}{2t}$$

$$(1+u)^m =$$

$$1 + \frac{m}{1!} u + \frac{m(m-1)}{2!} u^2 + \frac{m(m-1)(m-2)}{3!} u^3 + \dots$$

$$m = \frac{1}{2}$$

$$u = -4t$$

$$1 + 2t + 5t^2 + 14t^3 + 42t^4 + \dots + c_n t^n$$

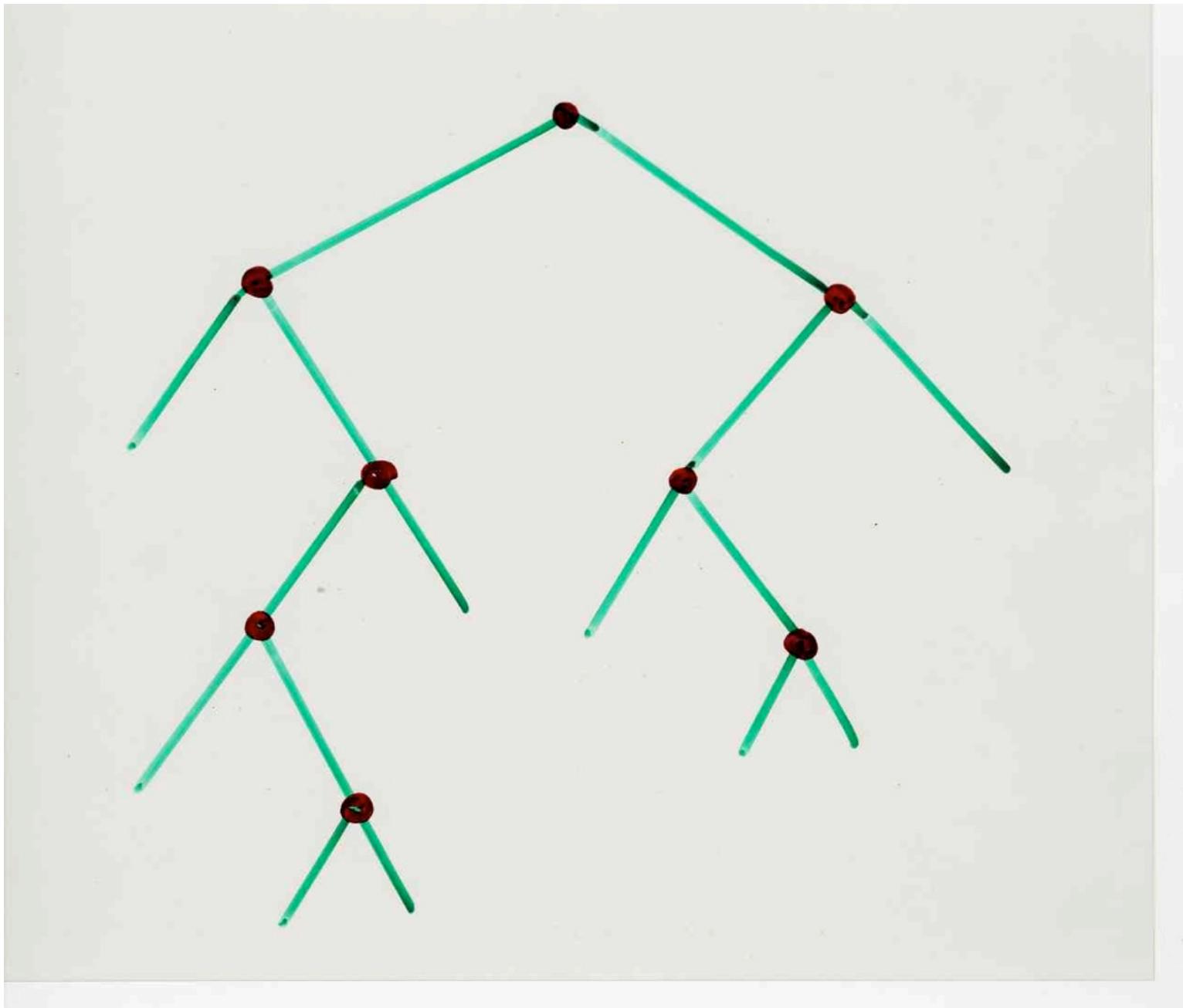
$$y = 1 + 2t + 5t^2 + 14t^3 + 42t^4 + \dots + C_n t^n + \dots$$

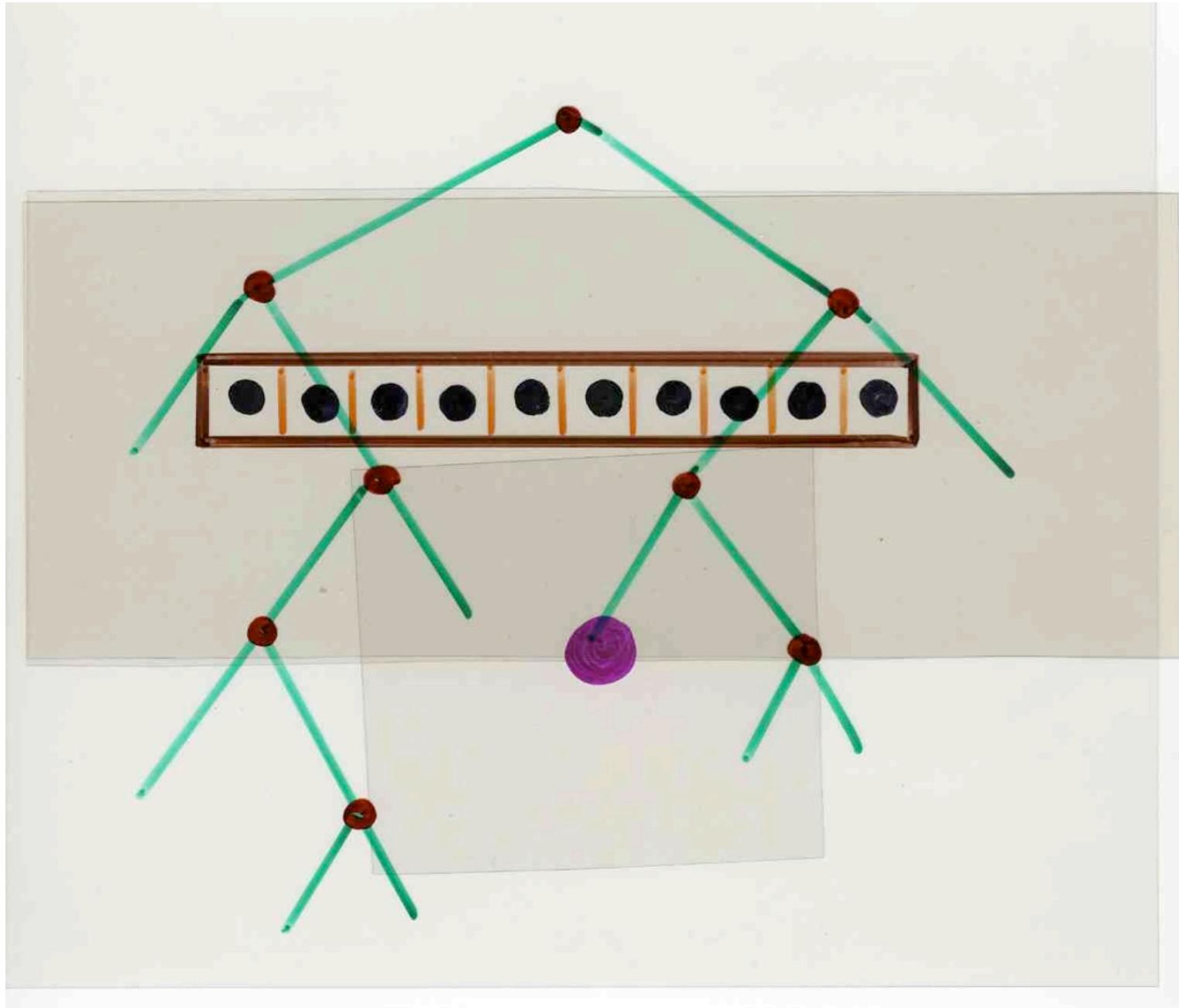
Will corde à
linge

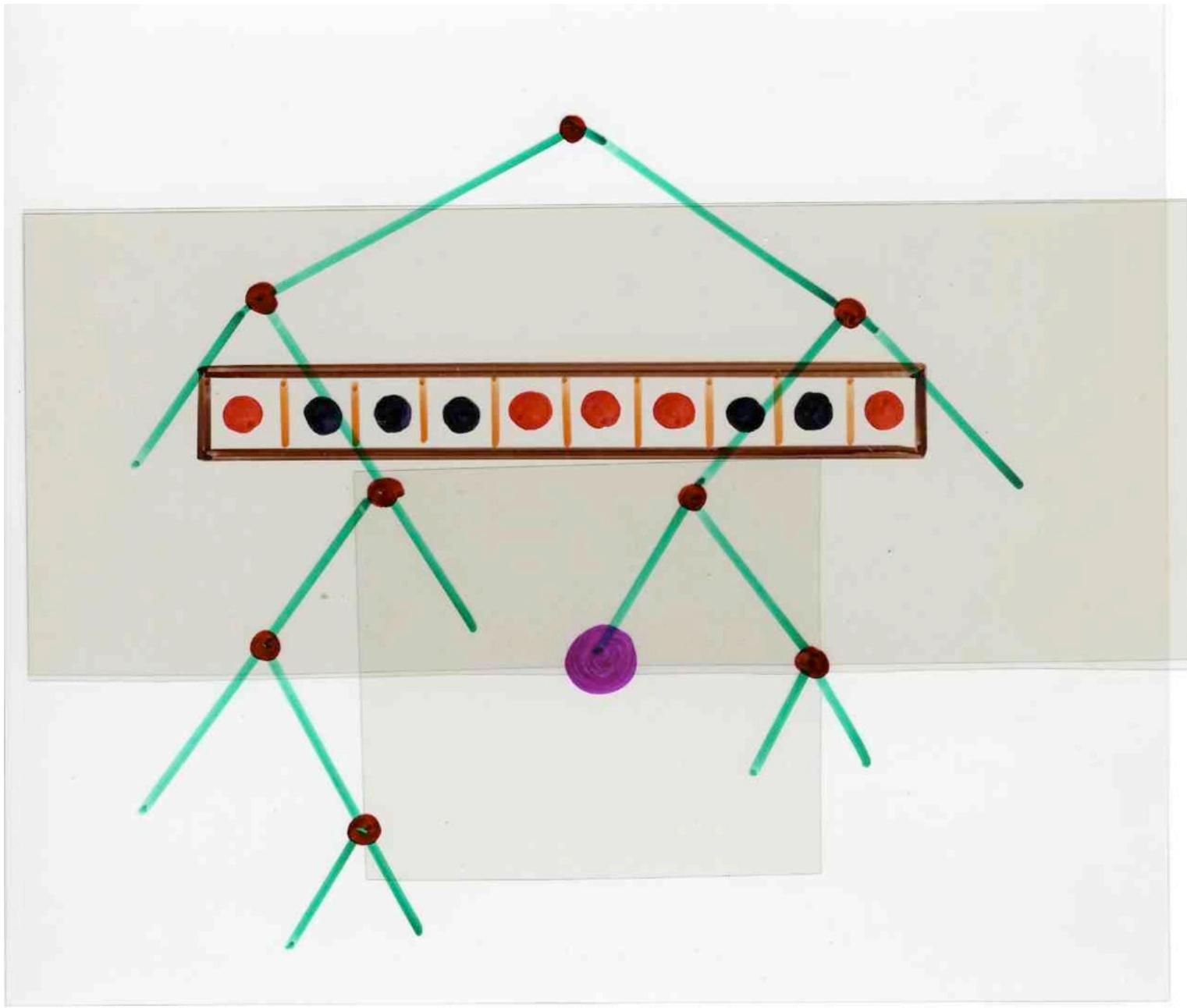
Combinatoire
bijective

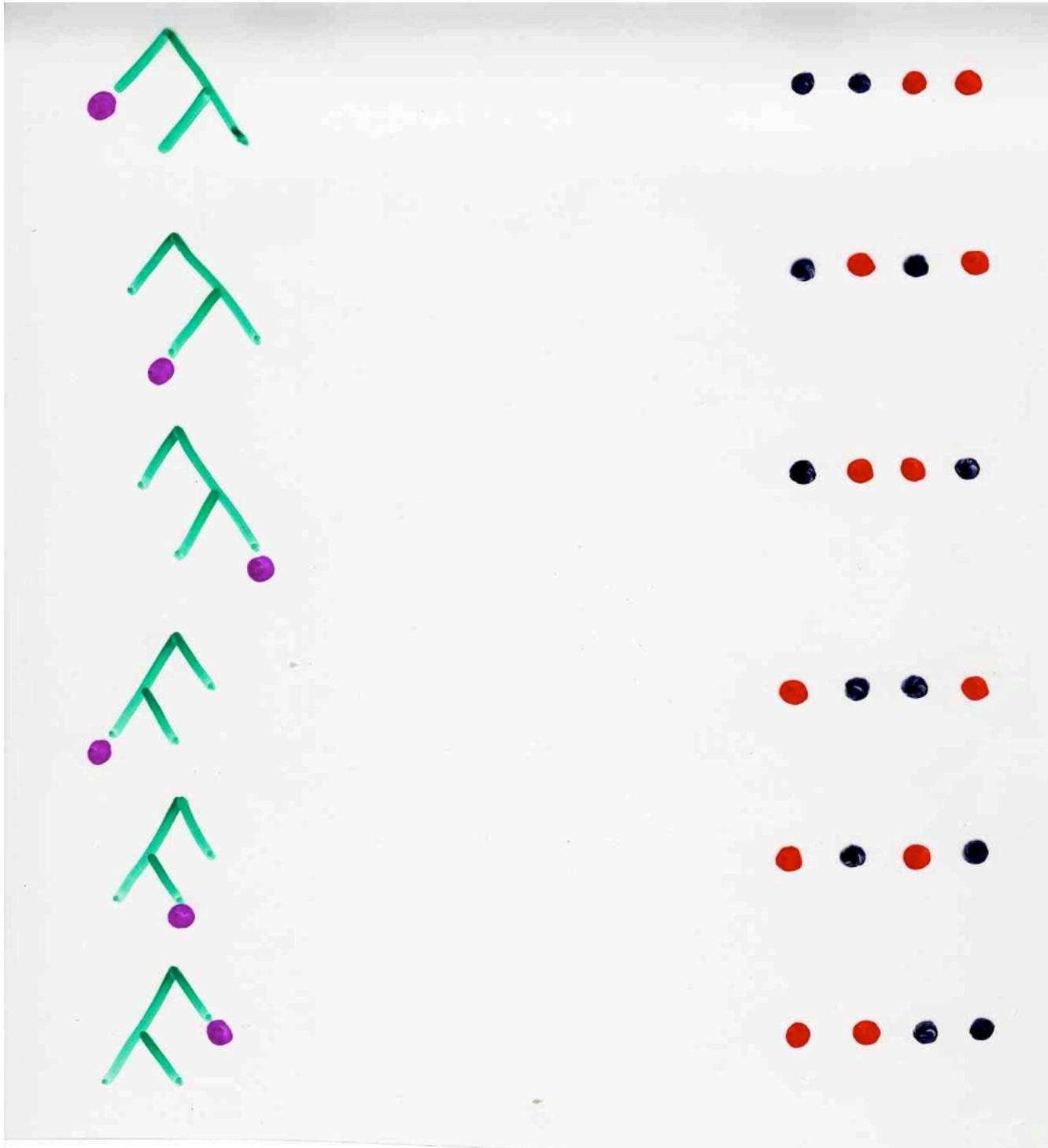
$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

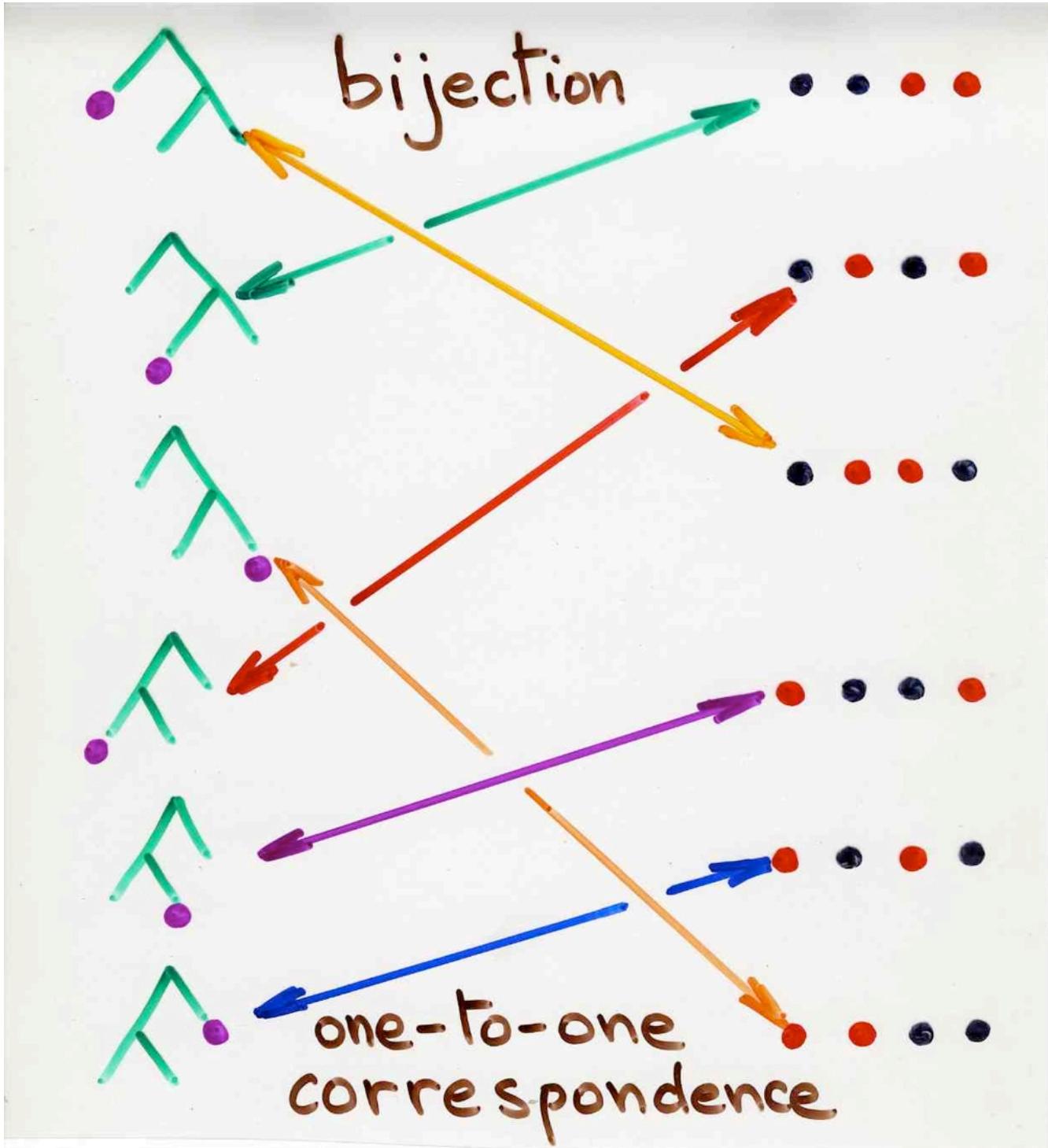
$$(n+1) C_n = \binom{2n}{n}$$











$S_{n,k} =$ nombre d'arbres binaires
ayant n sommets internes
et nombre de Strahler
égal à k

$$S_k = \sum_{n \geq 0} S_{n,k} t^n$$

$$S_1 = 1$$

$$S_2 = \frac{t}{1 - 2t}$$

$$S_3 = \frac{t^3}{1 - 6t + 10t^2 - 4t^3}$$

$$S_1 = 1$$

$$S_2 = \frac{t}{1 - 2t}$$

$$S_3 = \frac{t^3}{1 - 6t + 10t^2 - 4t^3}$$

$$S_4 = \frac{t^7}{1 - 14t + 78t^2 - 220t^3 + 330t^4 - 252t^5 + 84t^6 - 8t^7}$$

1 , 3 , 21 , 987 , ...

1, 2, 3, 5, 8, 13, 21, 34, 55,

89, 144, 233, 377, 610, 987,

① 2, ③ 5, 8, 13, 21, 34, 55,
89, 144, 233, 377, 610, 987,

① 1, 2, ③ 3, 5, 8, 13, ② 21, 34, 55,
 F_1 F_3 , F_7 , 89, 144, 233, 377, 610, 987,

Fibonacci ! F_{15}

$F_{2^k - 1}$

Pascal Triangle

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Pascal Triangle									
1	1								
1	2	1							
1	3	3	1						
1	4	6	4	1					
1	5	10	10	5	1				
1	6	15	20	15	6	1			
1	7	21	35	35	21	7	1		
1	8	28	56	70	56	28	8	1	
1	9	36	84	126	126	84	36	9	1
1	10	45	120	210	252	210	120	45	10
1	11	55	165	330	462	462	330	165	55
1	12	66	220	495	792	924	792	495	220
1	13	88	286	715	1287	1716	1716	1287	715
1	14	91	364	1001	2002	3003	3432	3003	2002
1	15	105	455	1365	3003	5005	6435	6435	5005

Pascal Triangle

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Pascal Triangle														
$\binom{n}{k} = \frac{n!}{k!(n-k)!}$														
1														
1	1													
1	2	1												
1	3	3	1											
1	4	6	4	1										
1	5	10	10	5	1									
1	6	15	20	15	6	1								
1	7	21	35	35	21	7	1							
1	8	28	56	70	56	28	8	1						
1	9	36	84	126	126	84	36	9	1					
1	10	45	120	210	252	210	120	45	10					
1	11	55	165	330	462	462	330	165	55					
1	12	66	220	495	792	924	792	495	220					
1	13	88	286	715	1287	1716	1716	1287	715					
1	14	91	364	1001	2002	3003	3432	3003	2002					
1	15	105	455	1365	3003	5005	6435	6435	5005					

Combinatorics
is
beautiful



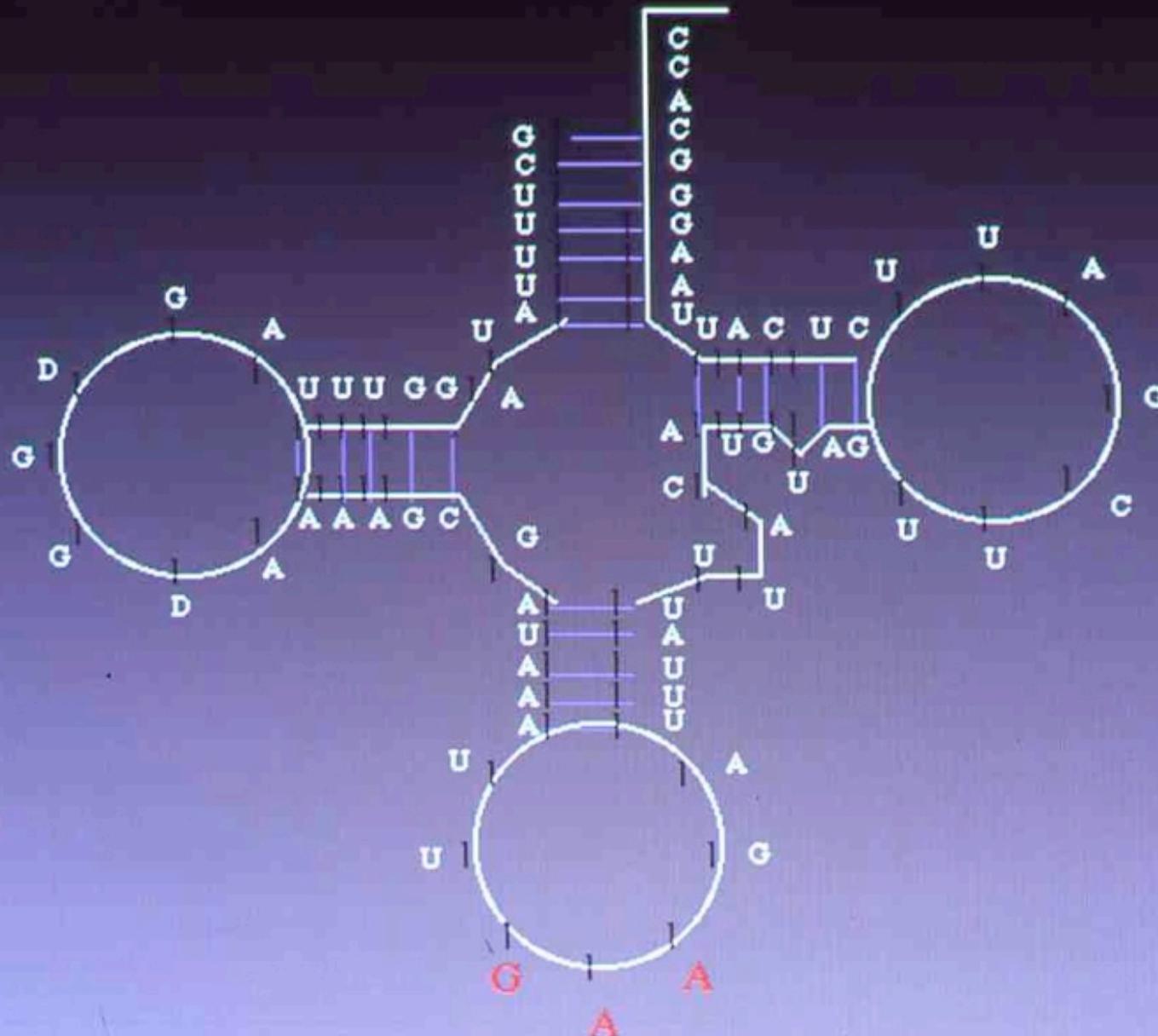
Aerial view of a forest with a grid pattern of paths or roads.

6

Structures
secondaires
en
biologie
moléculaire

ARNt^{Phe}

Phenylalanine



Adenine
Uracyle
Guanine
Cytosine

Biologie moléculaire

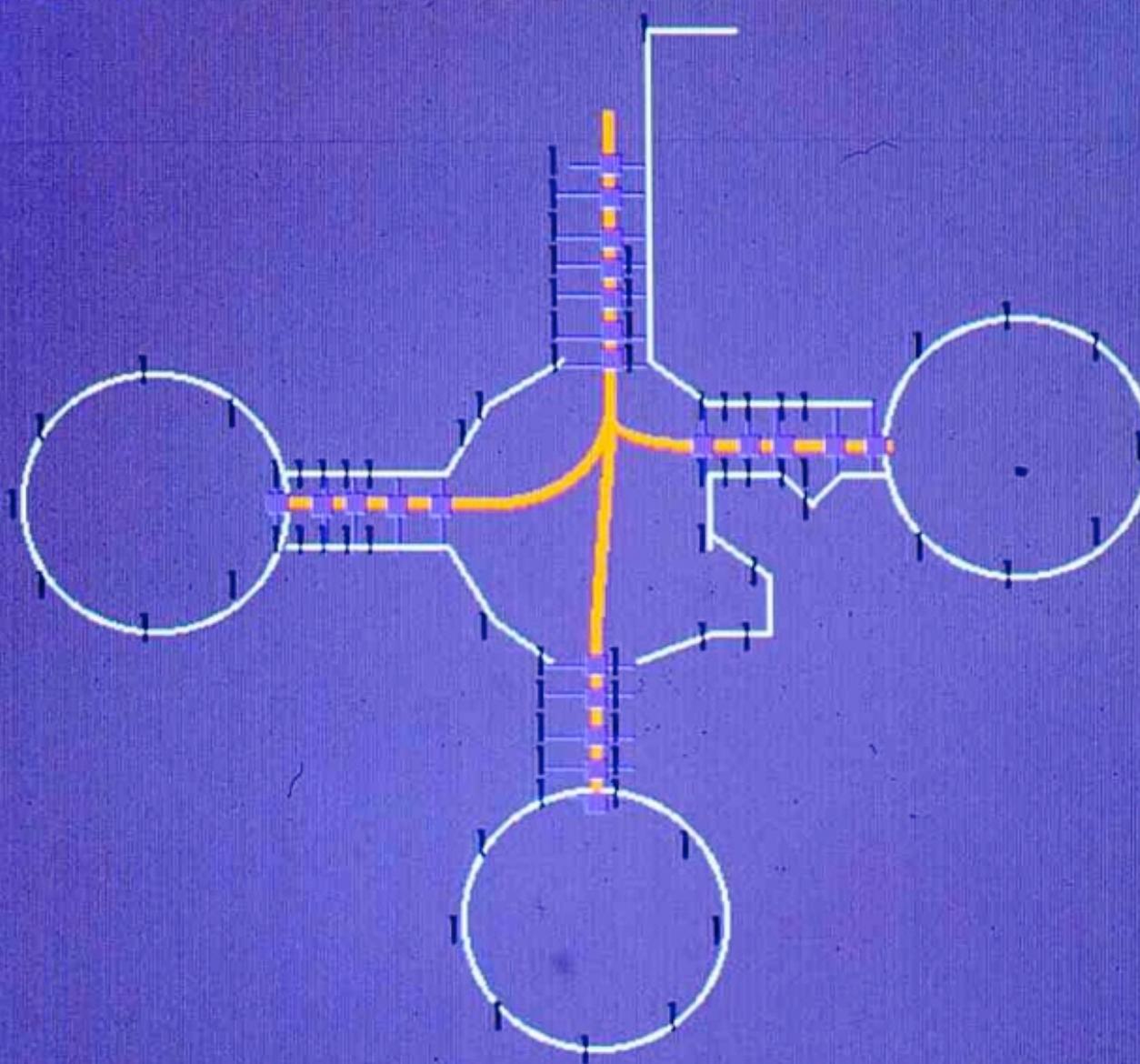
Structures secondaires d'acides nucléiques simple-brins

(ARN, ARN_t, ARN_m,...)

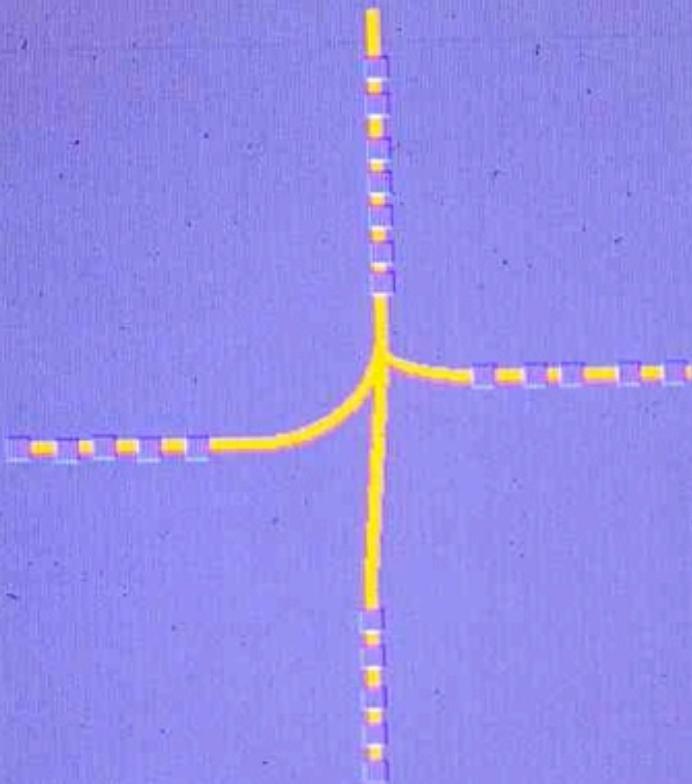
- Prédiction de la structure secondaire
(à partir de la structure primaire)
- homologies

ARNt^{Phe}

Phenylalanine



ARNt



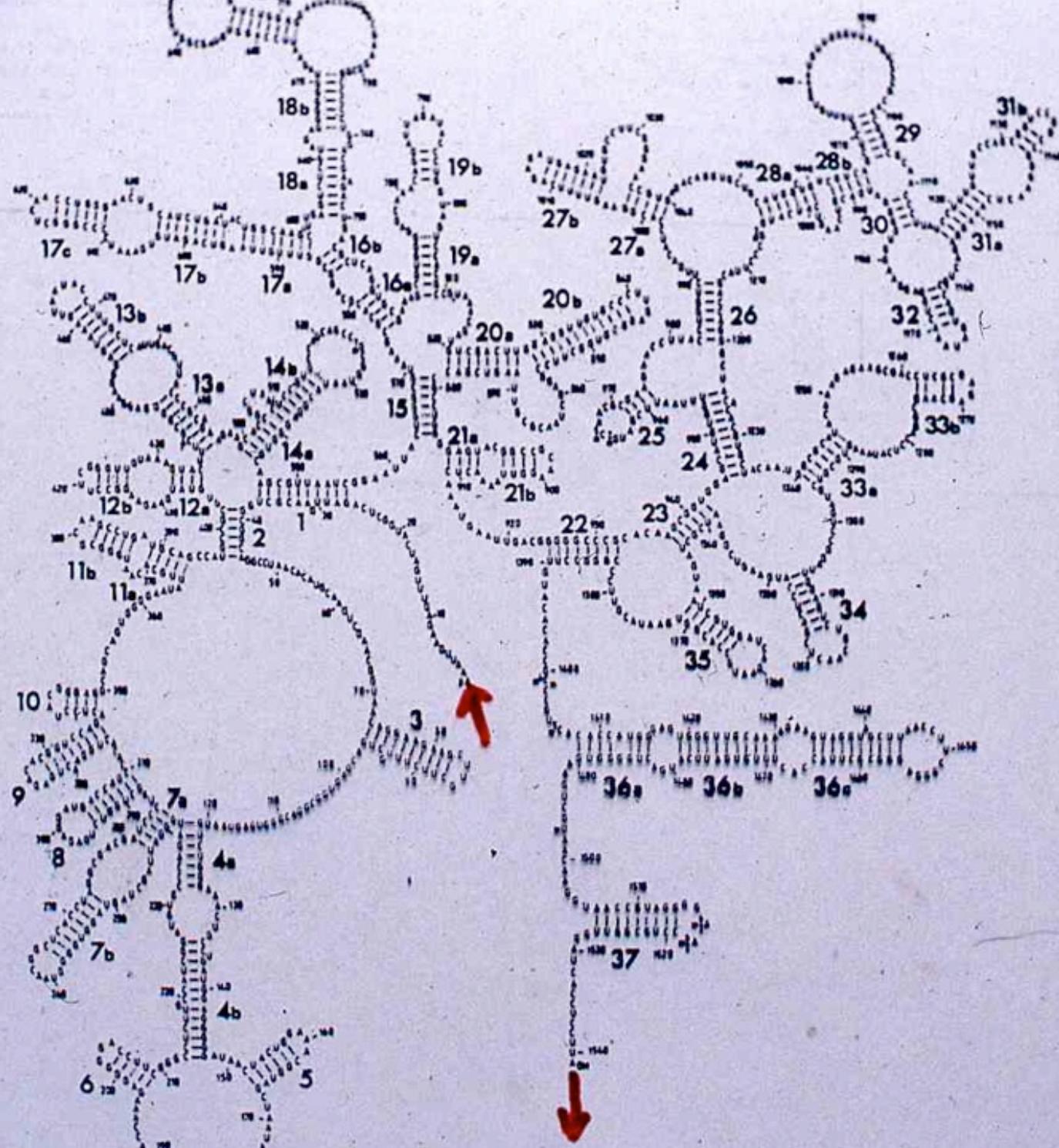
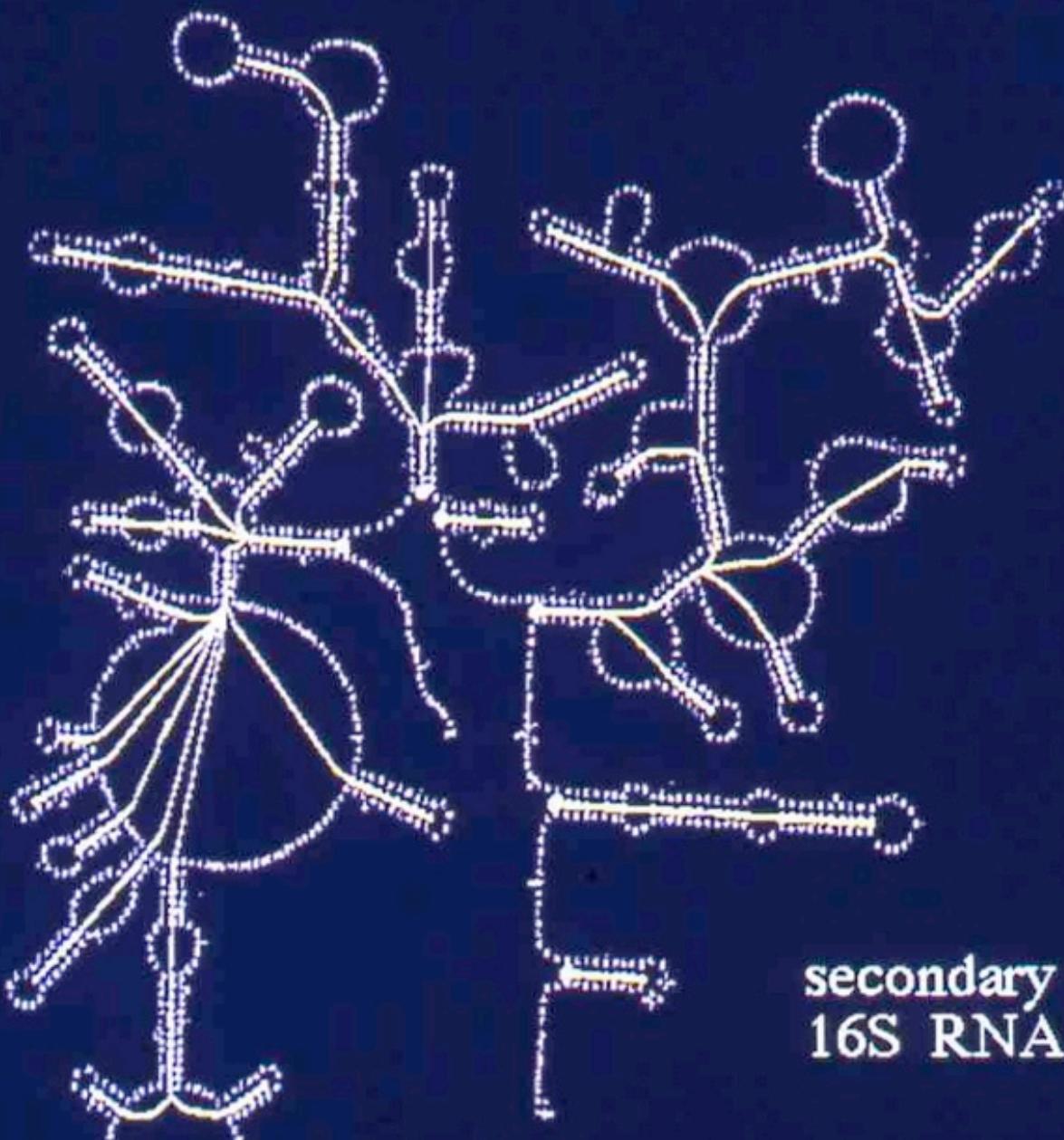
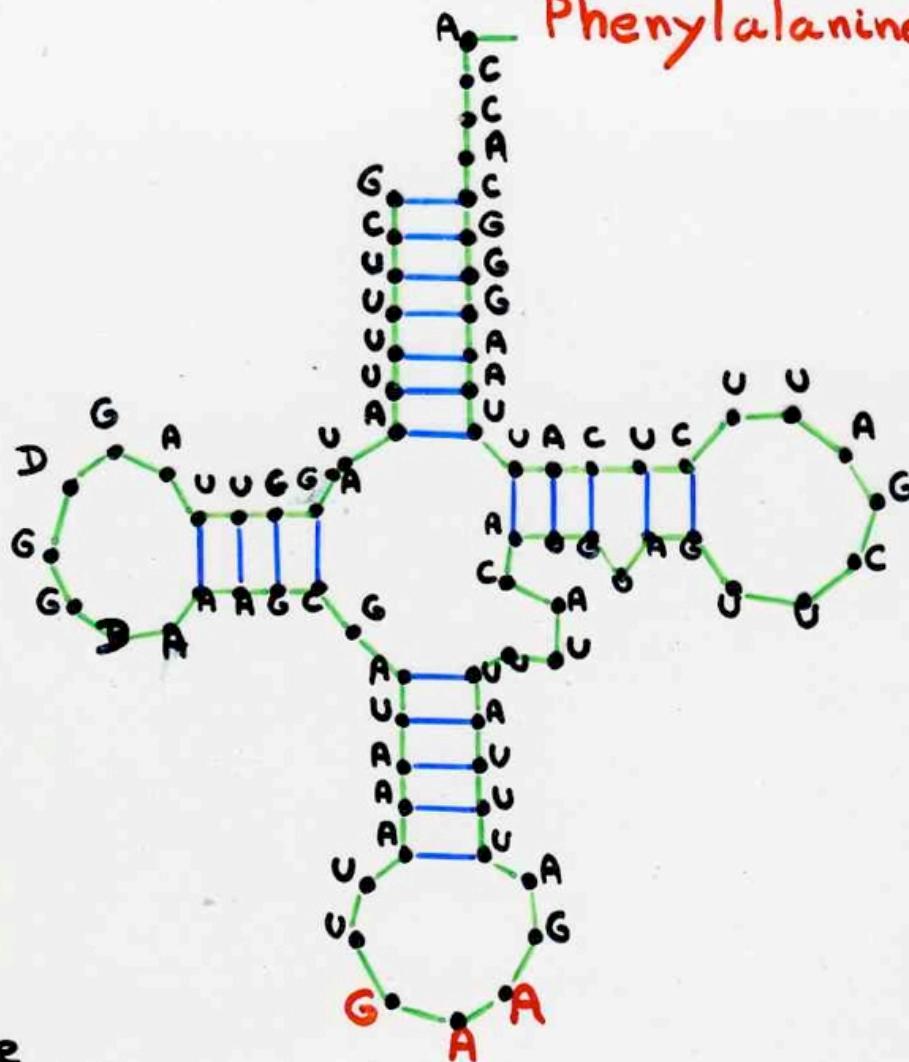


Fig. 1. Secondary structure model of the 16S rRNA from *E. coli*. This model has been fully described elsewhere [18]. The various secondary structure motifs



secondary structure
16S RNA *E.Coli*

Phenylalanine



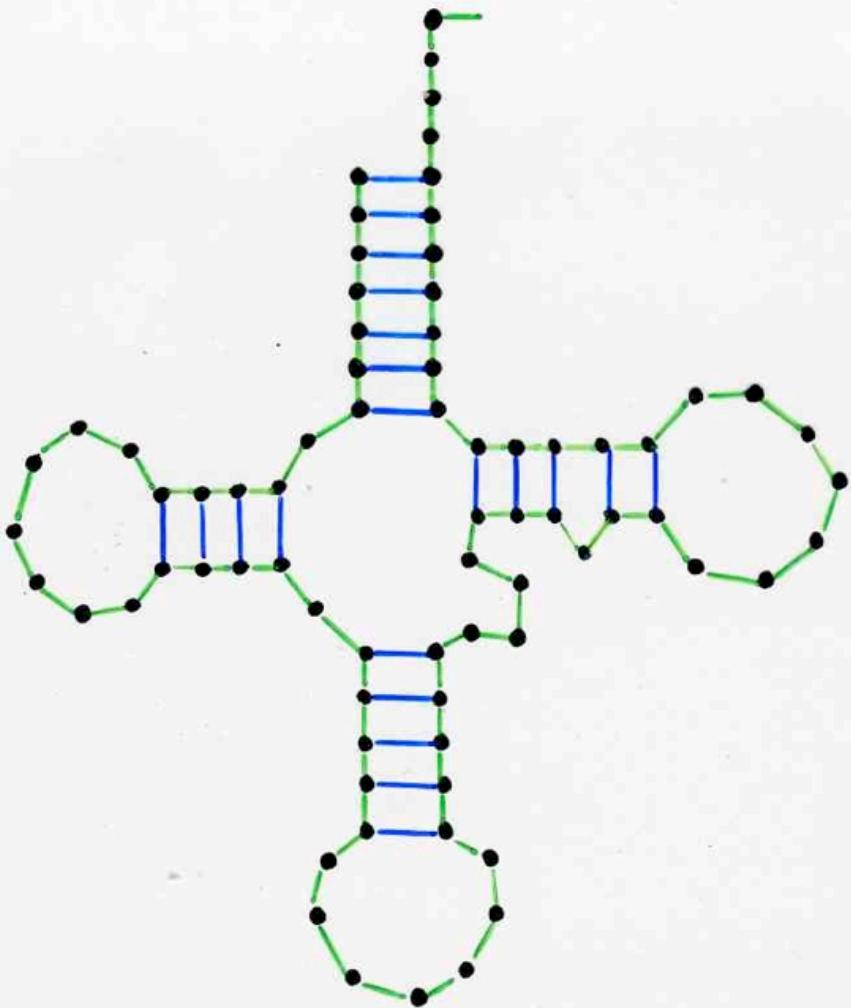
A adenine

U uracile

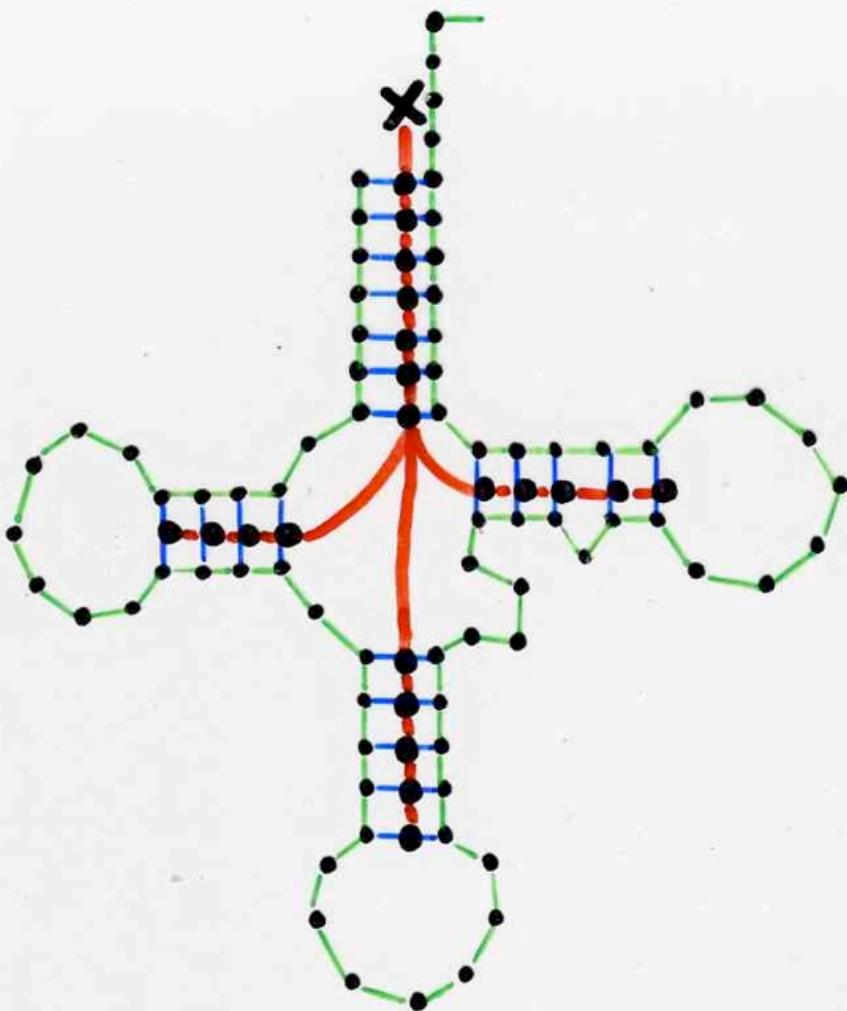
G Guanine

C cytosine

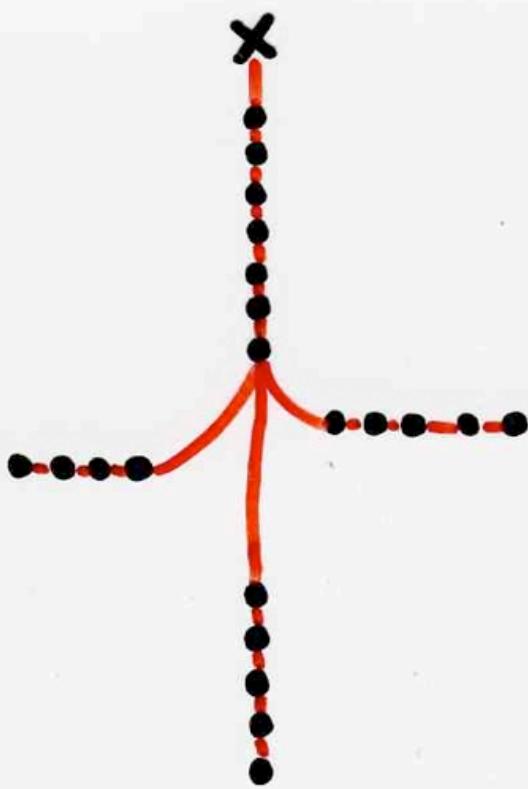
t ARN^{Phe}

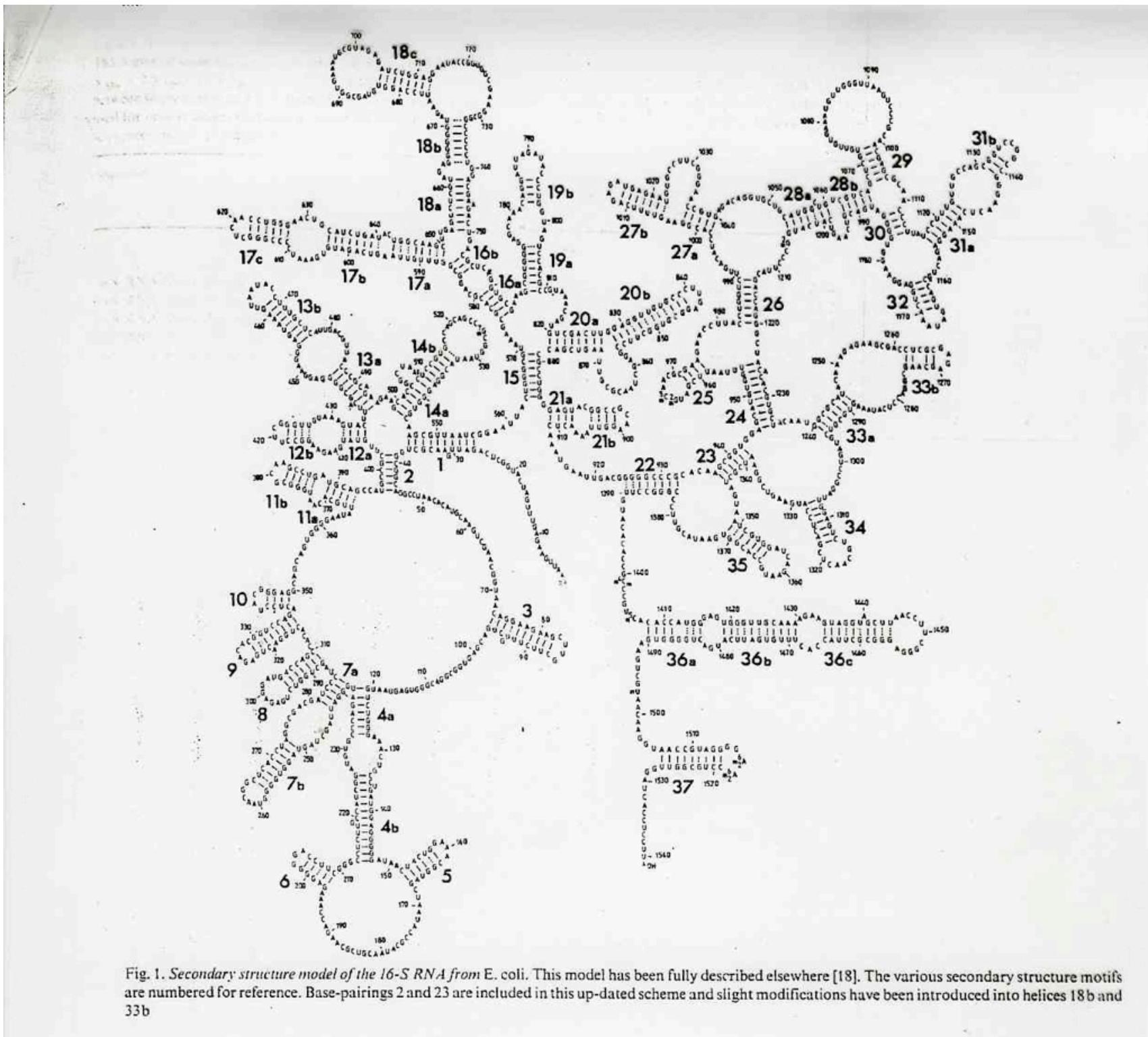


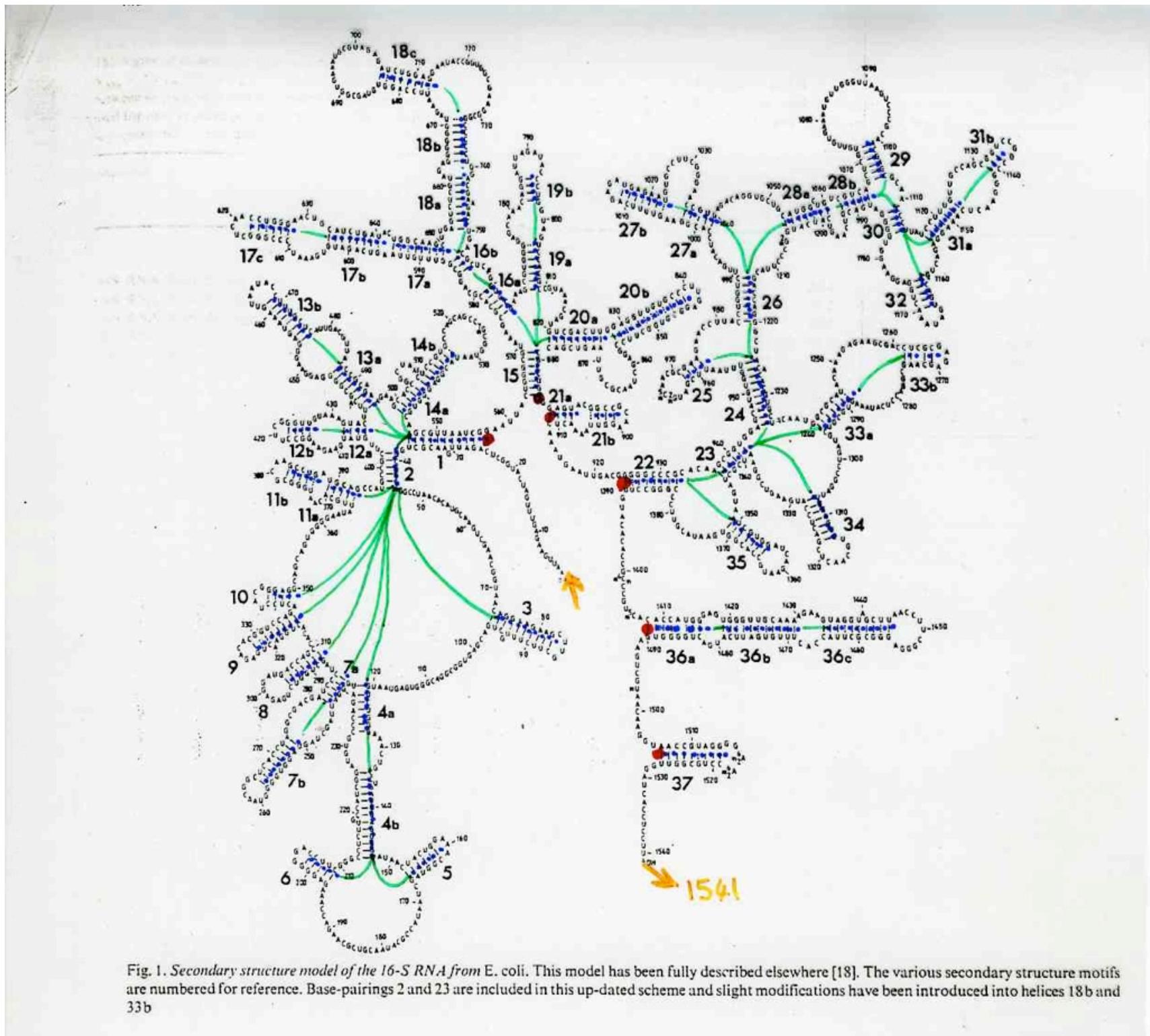
tARN^{Phe}



tARN^{Phe}

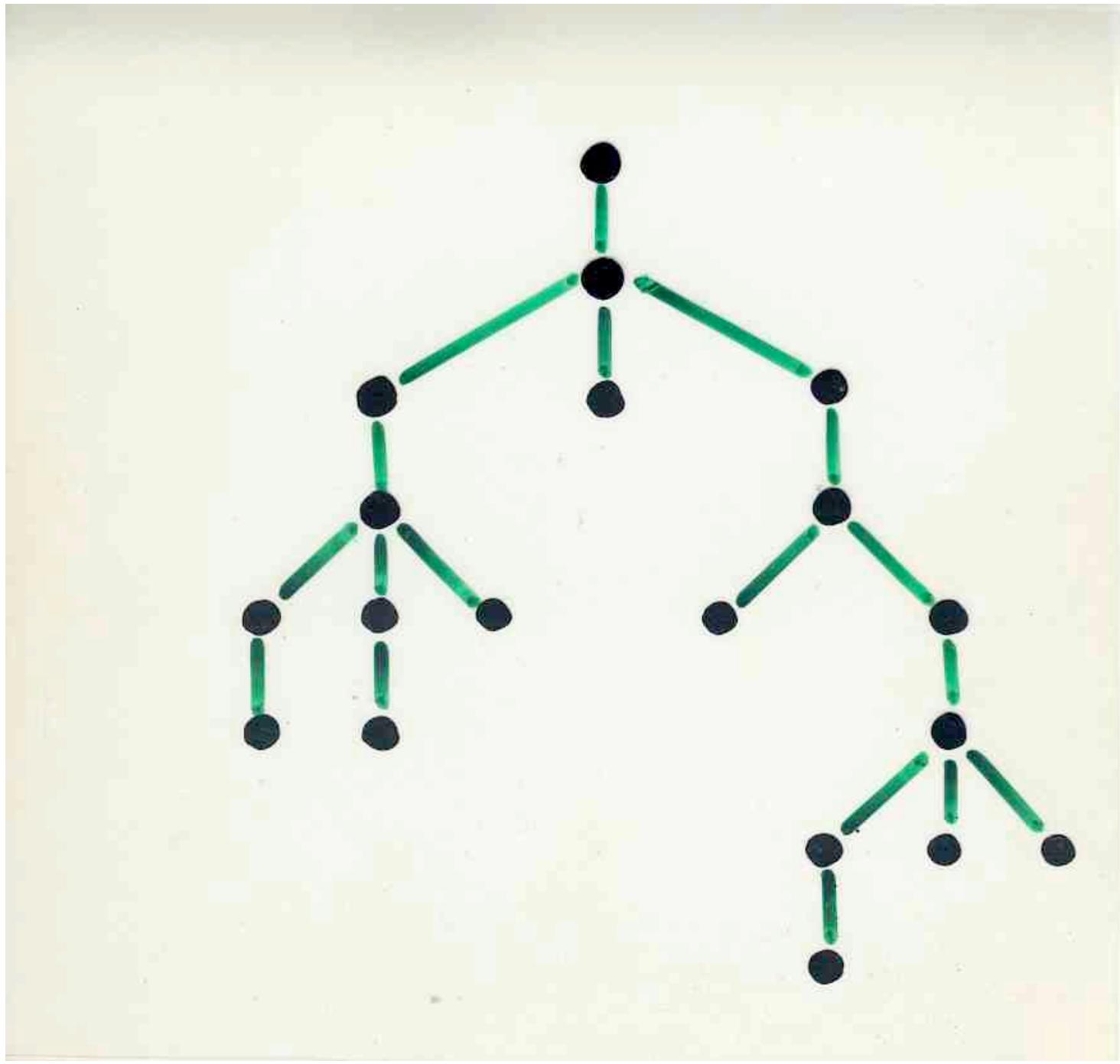


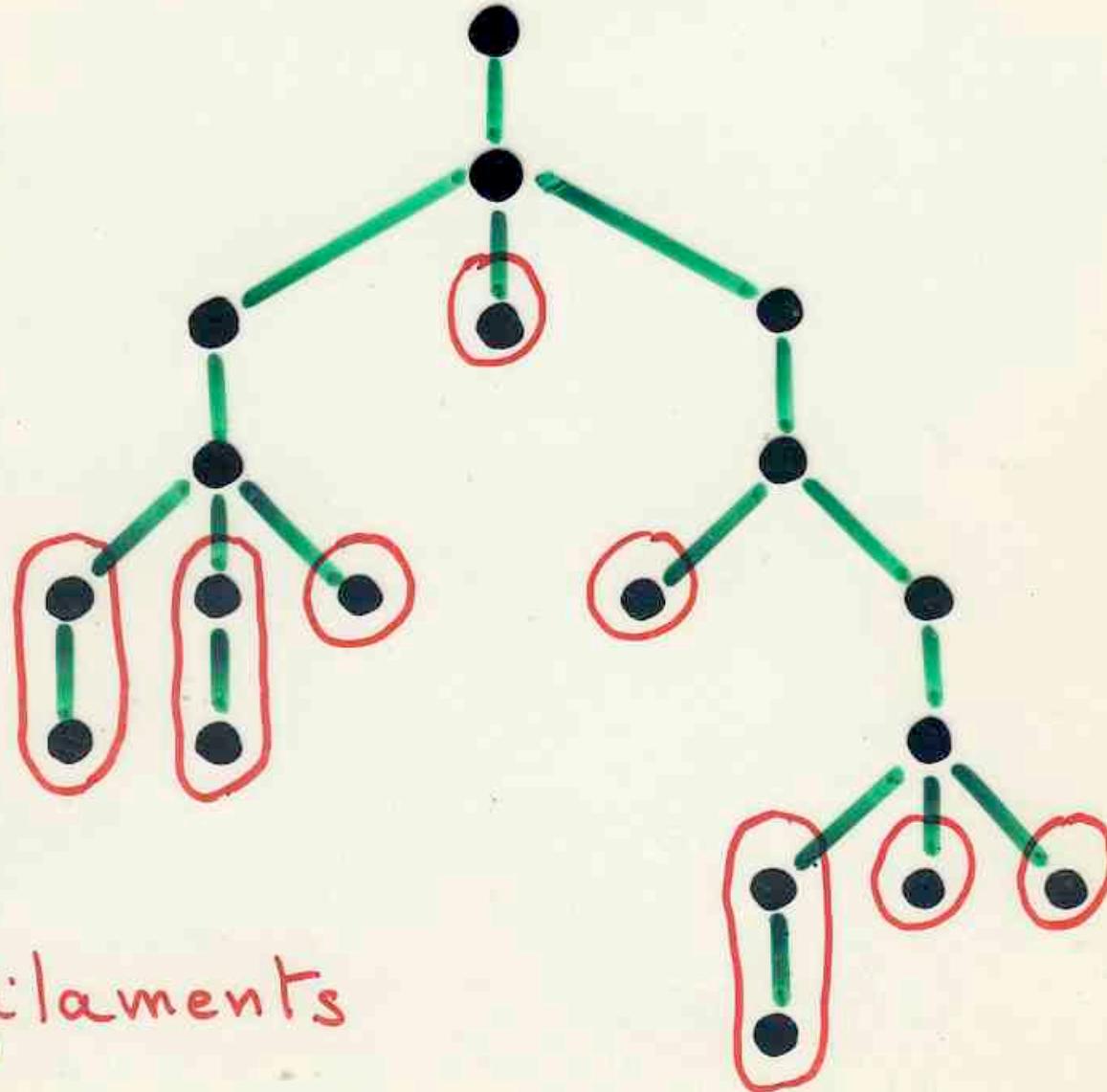




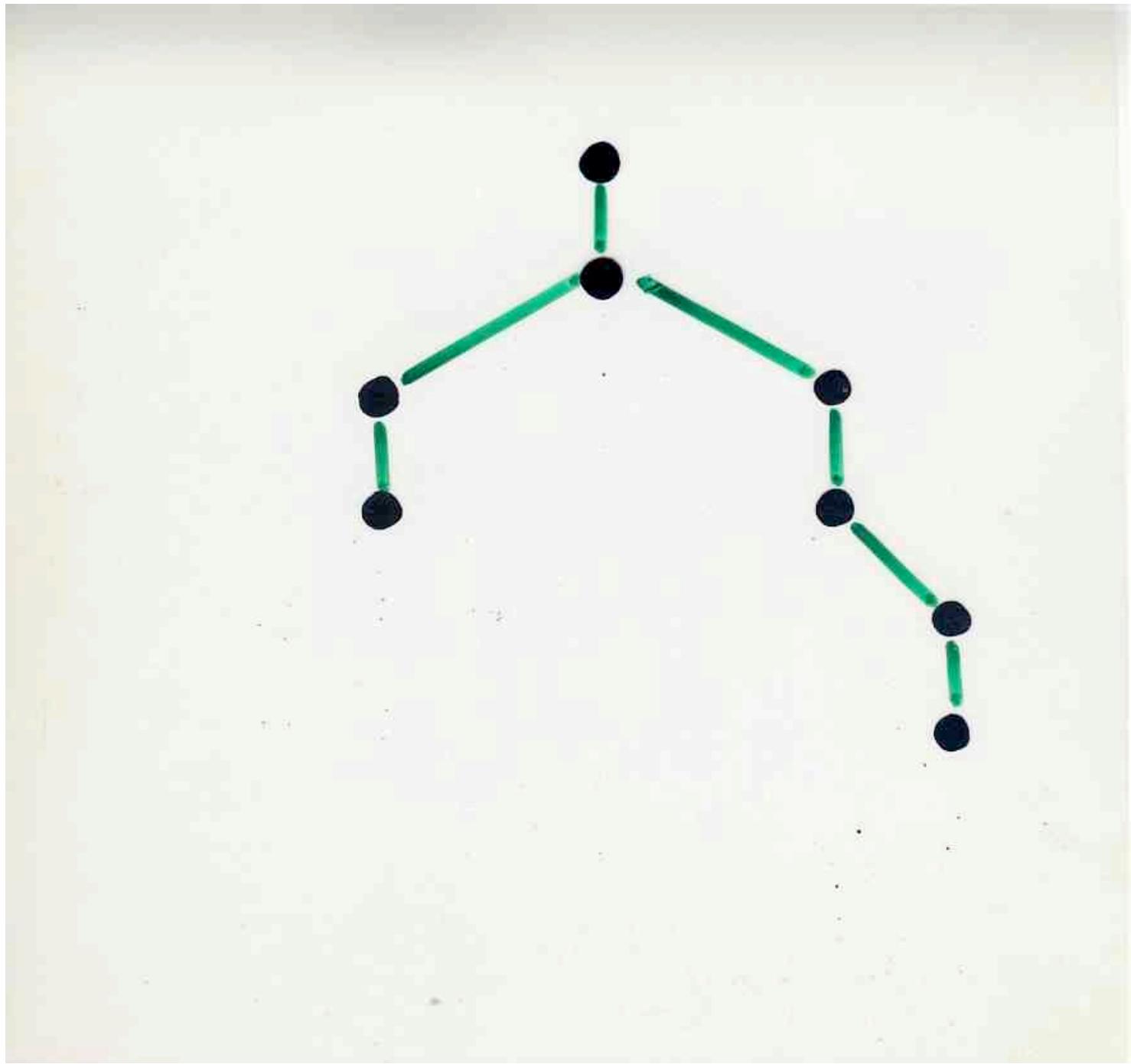
M. Waterman

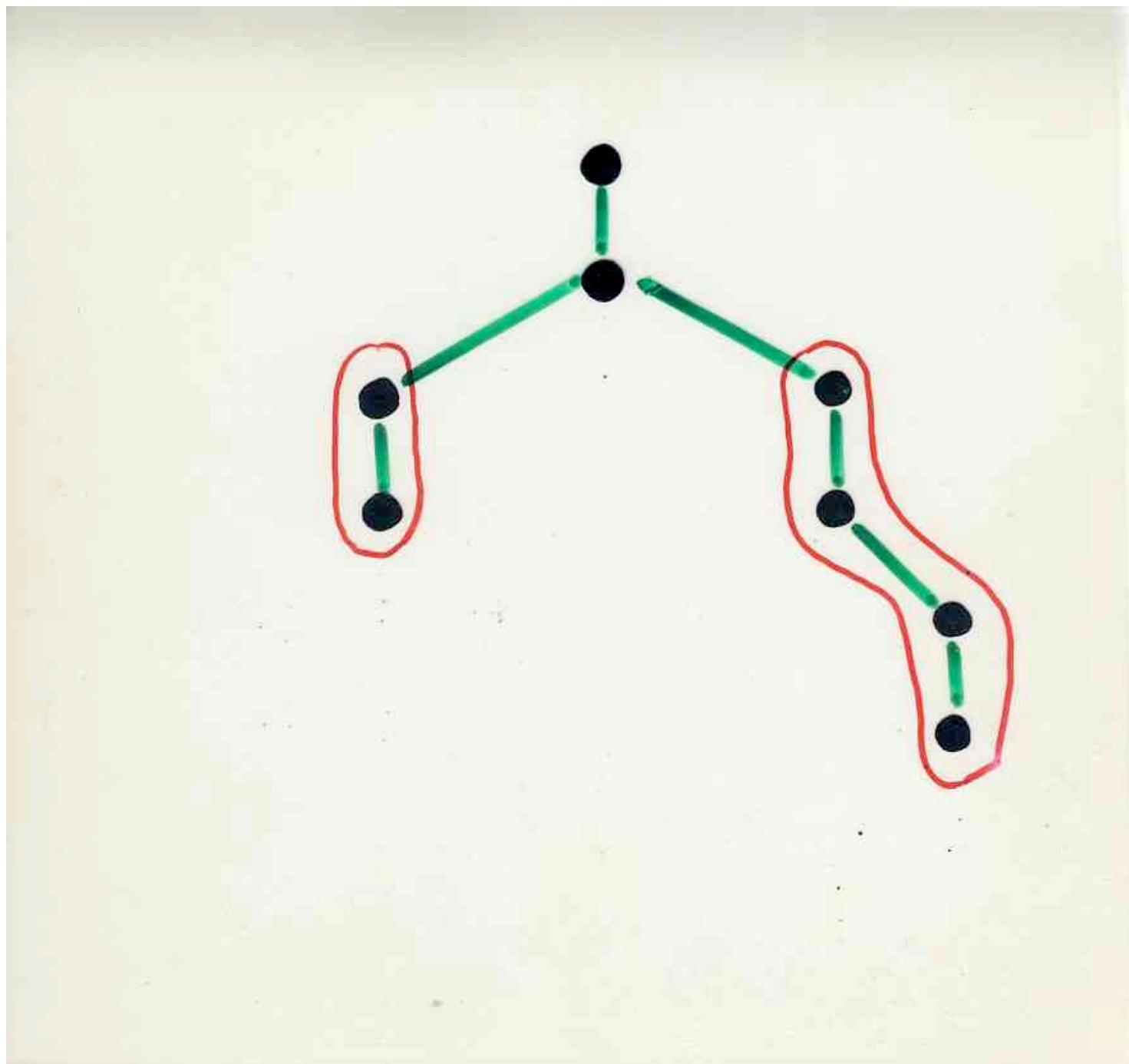
- ordre d'une structure secondaire ARN
- ordre de la forêt sous-jacente
- ordre d'un arbre (planaire)

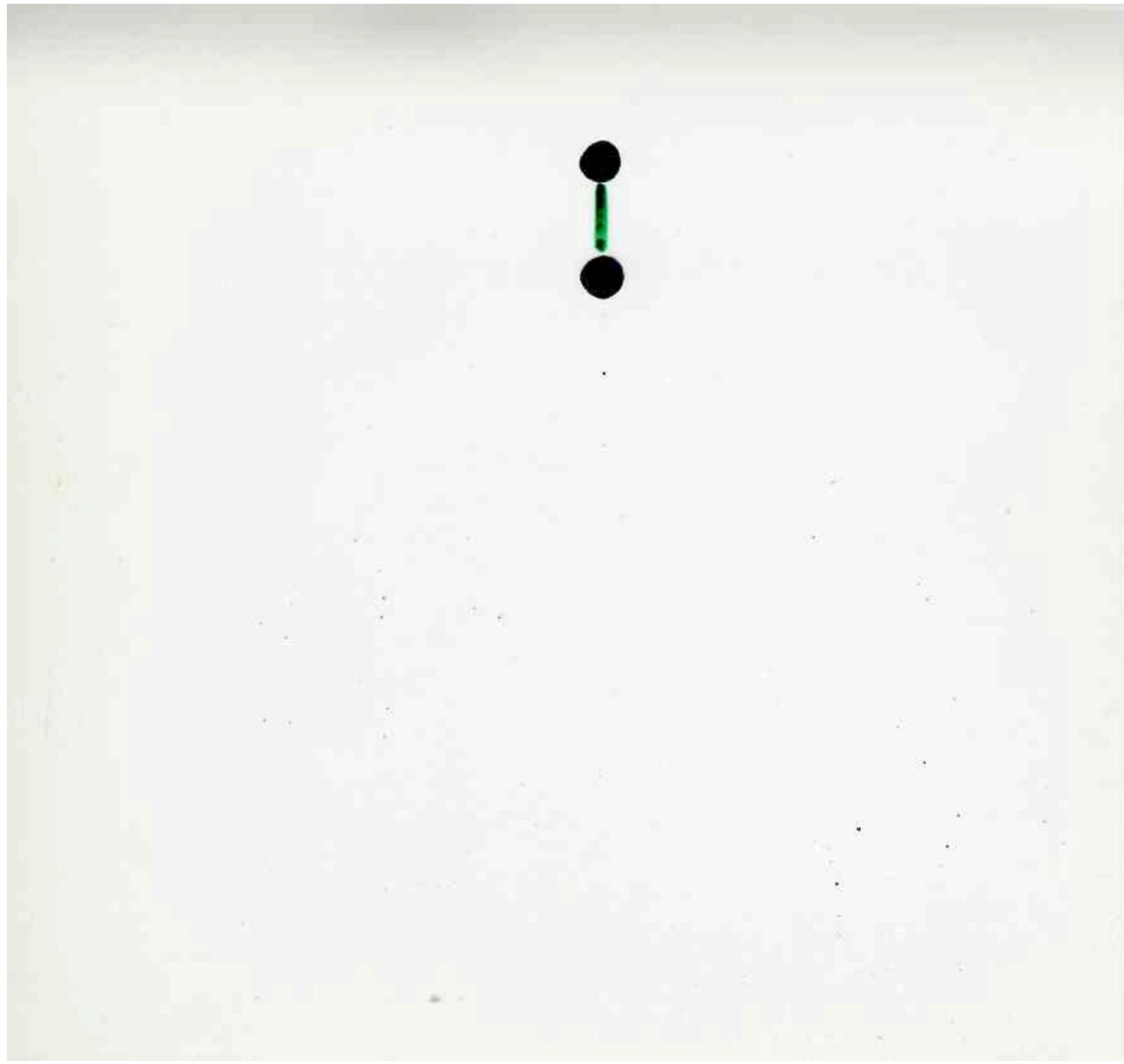


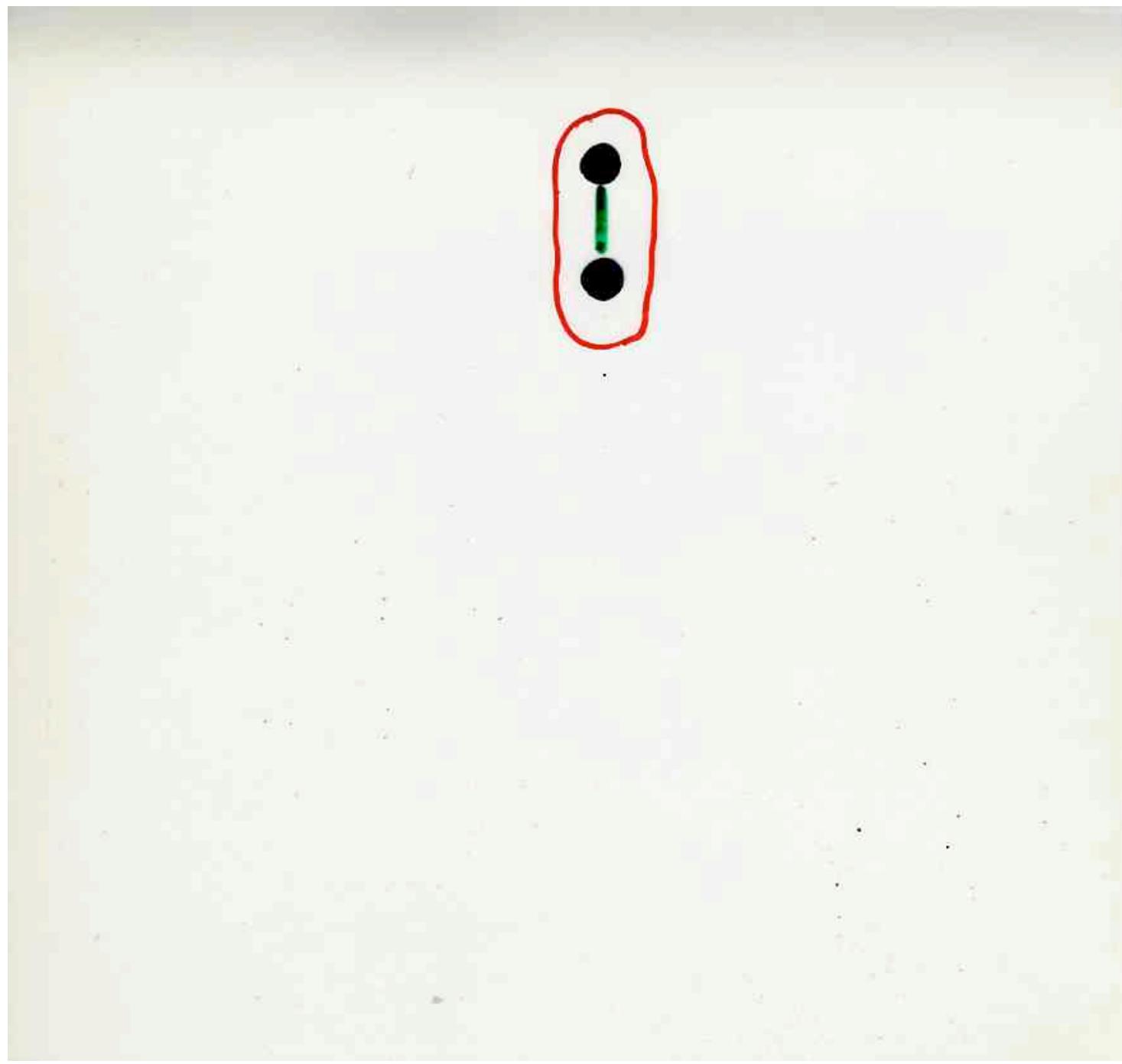


filaments









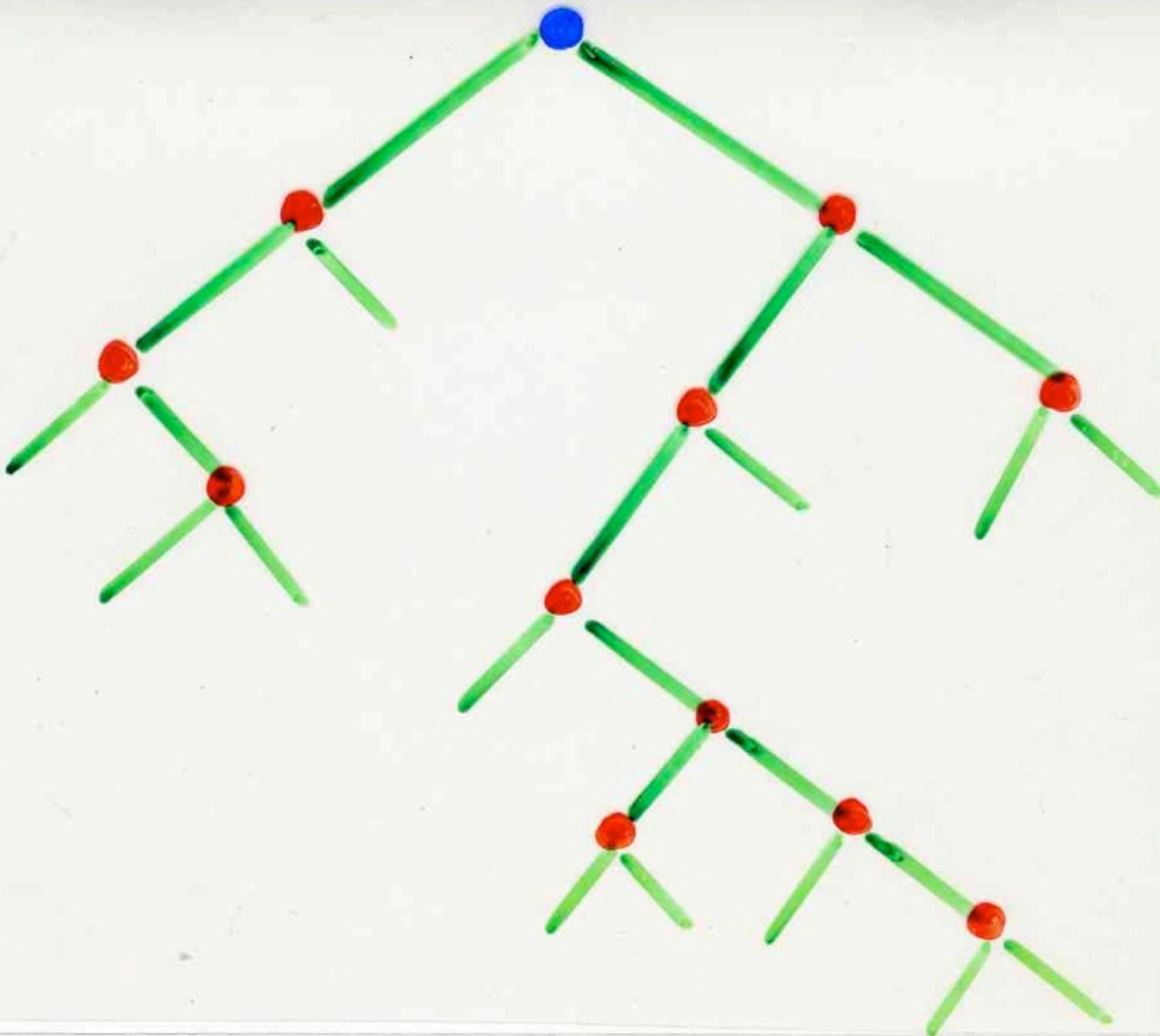
$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k

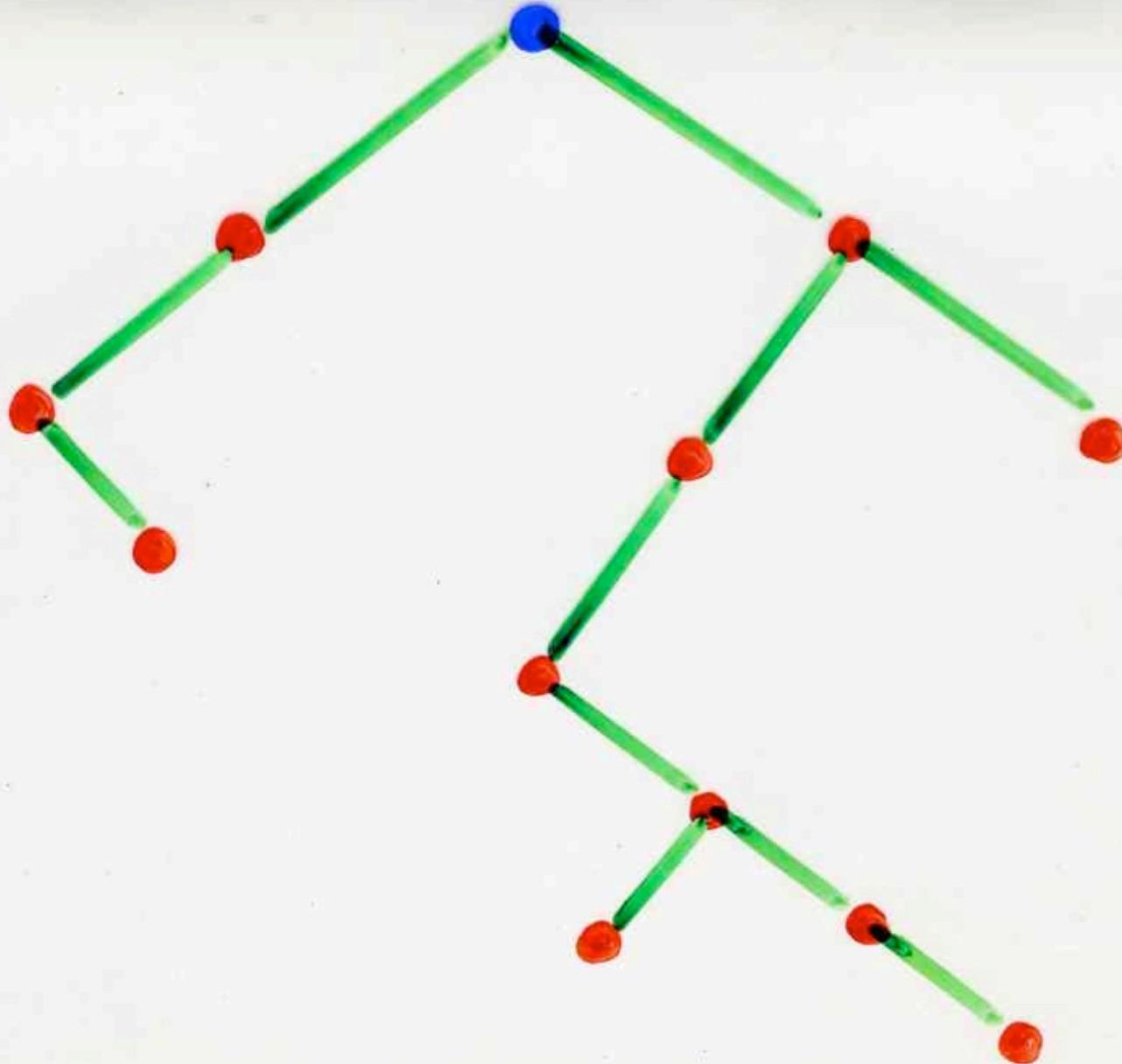
$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k

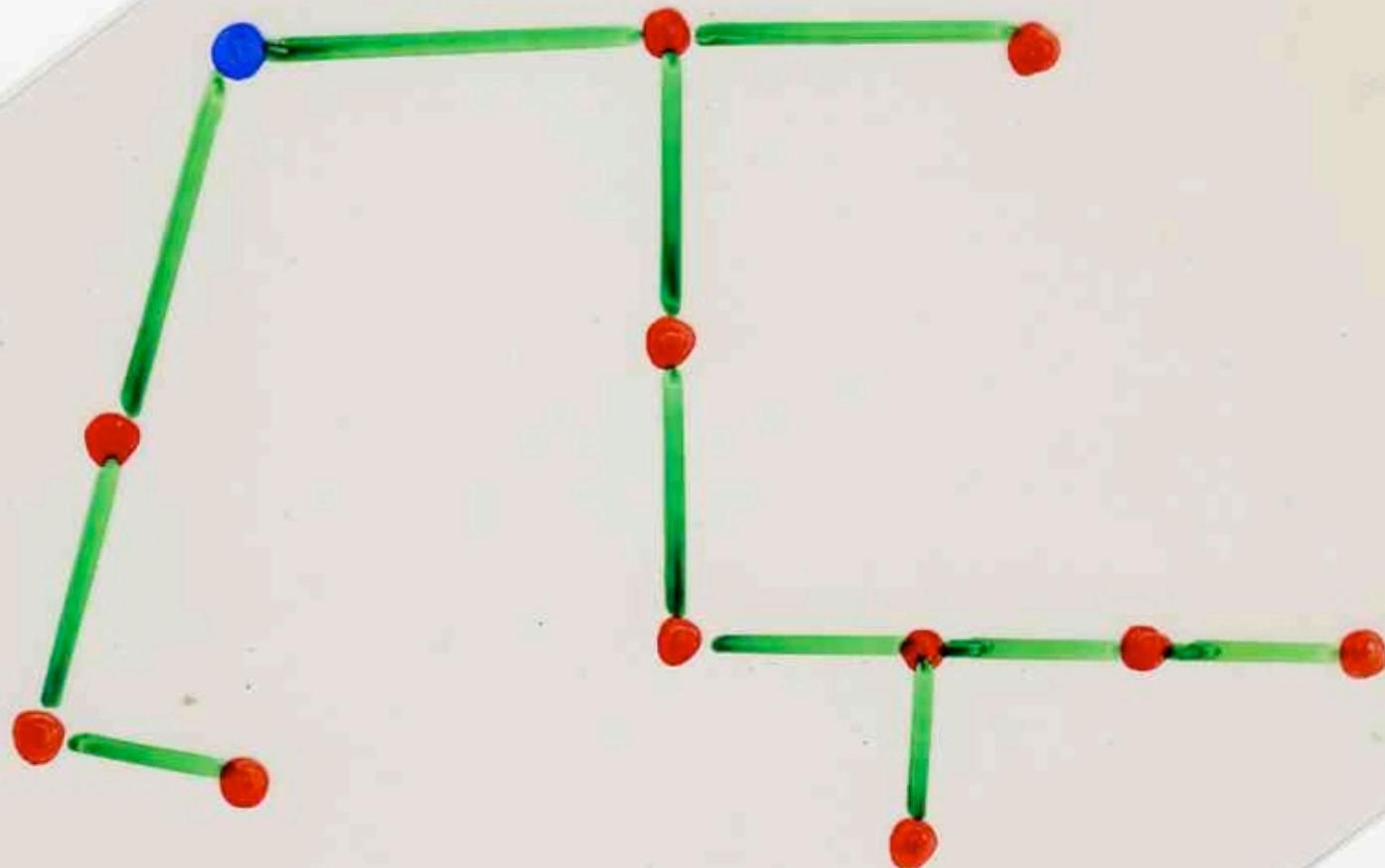
$$= S_{n,k}$$

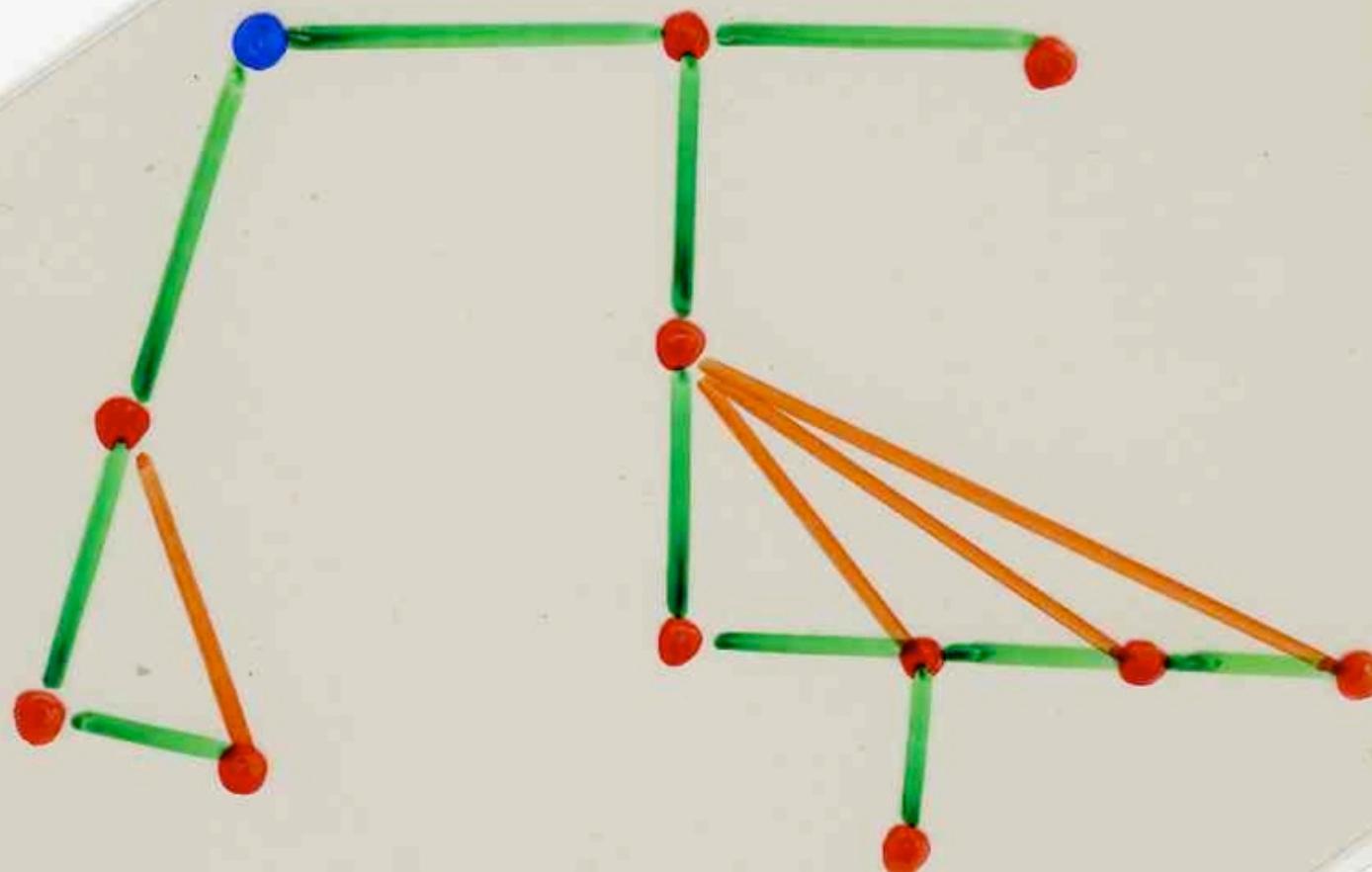
Vauclusade de Chaumont
X. V. (1985) (2001)

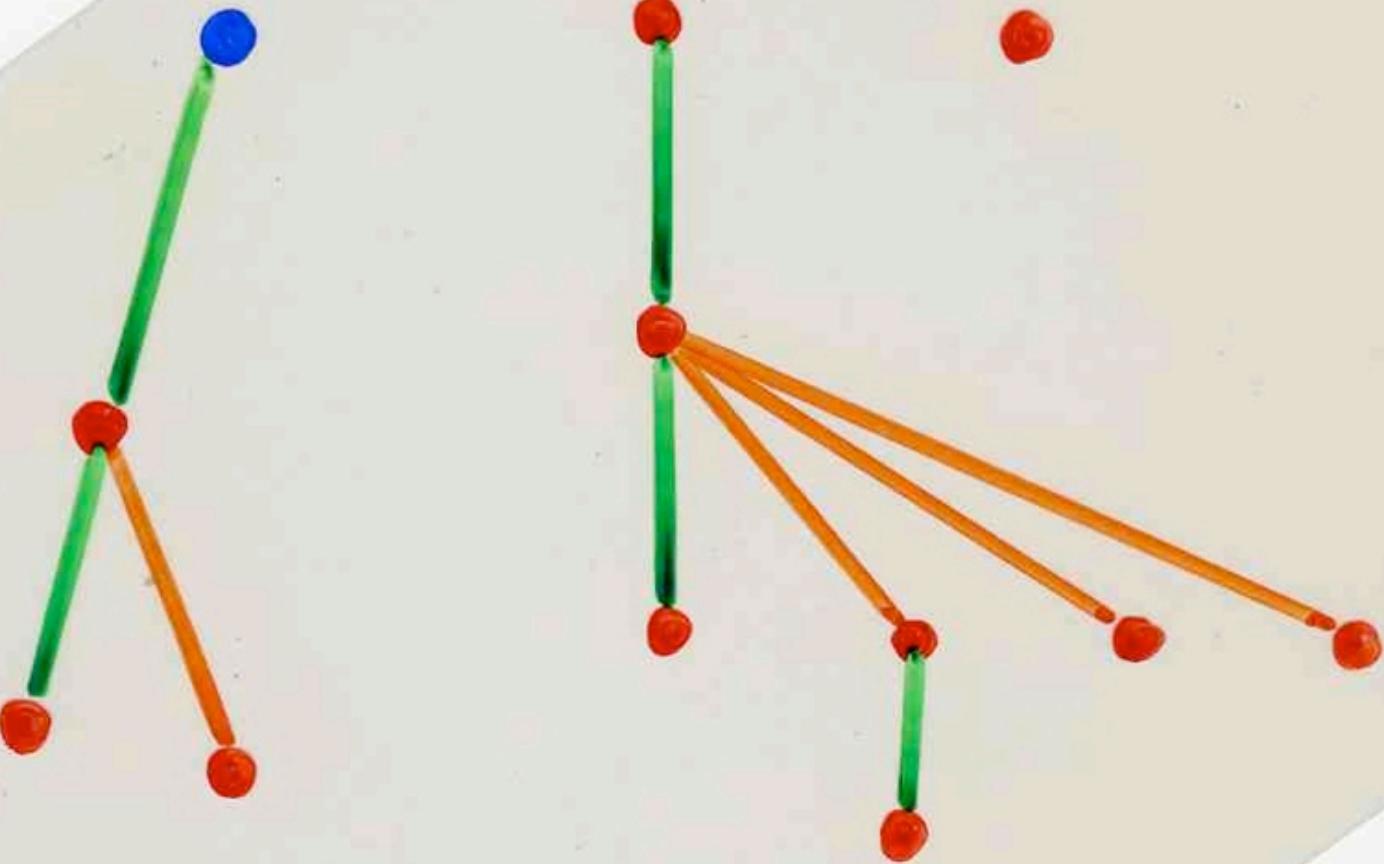
D. Zeilberger (1985)

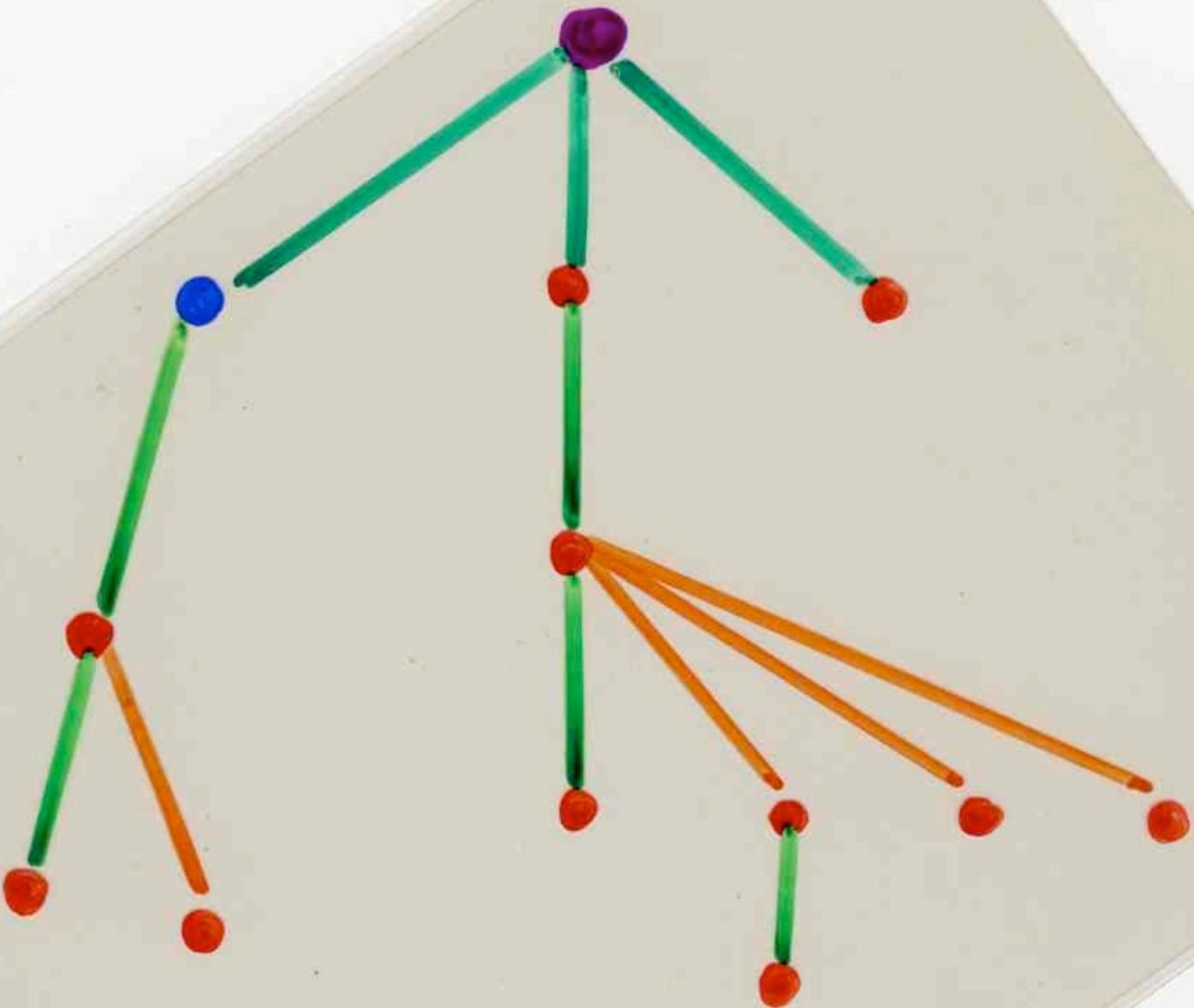








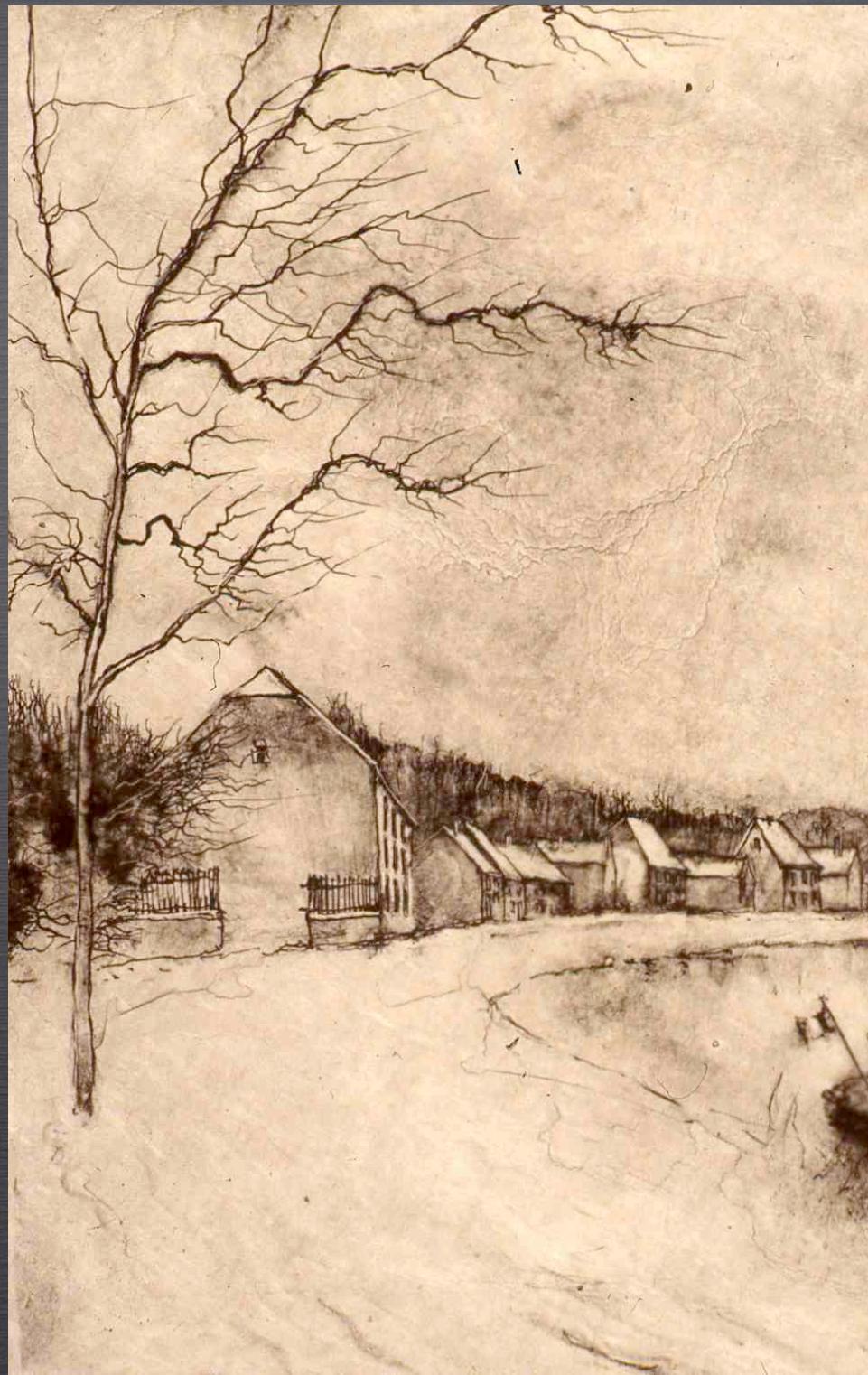






7

Matrices de ramification





fractals

- auto-similarité
- dimension fractionnaire

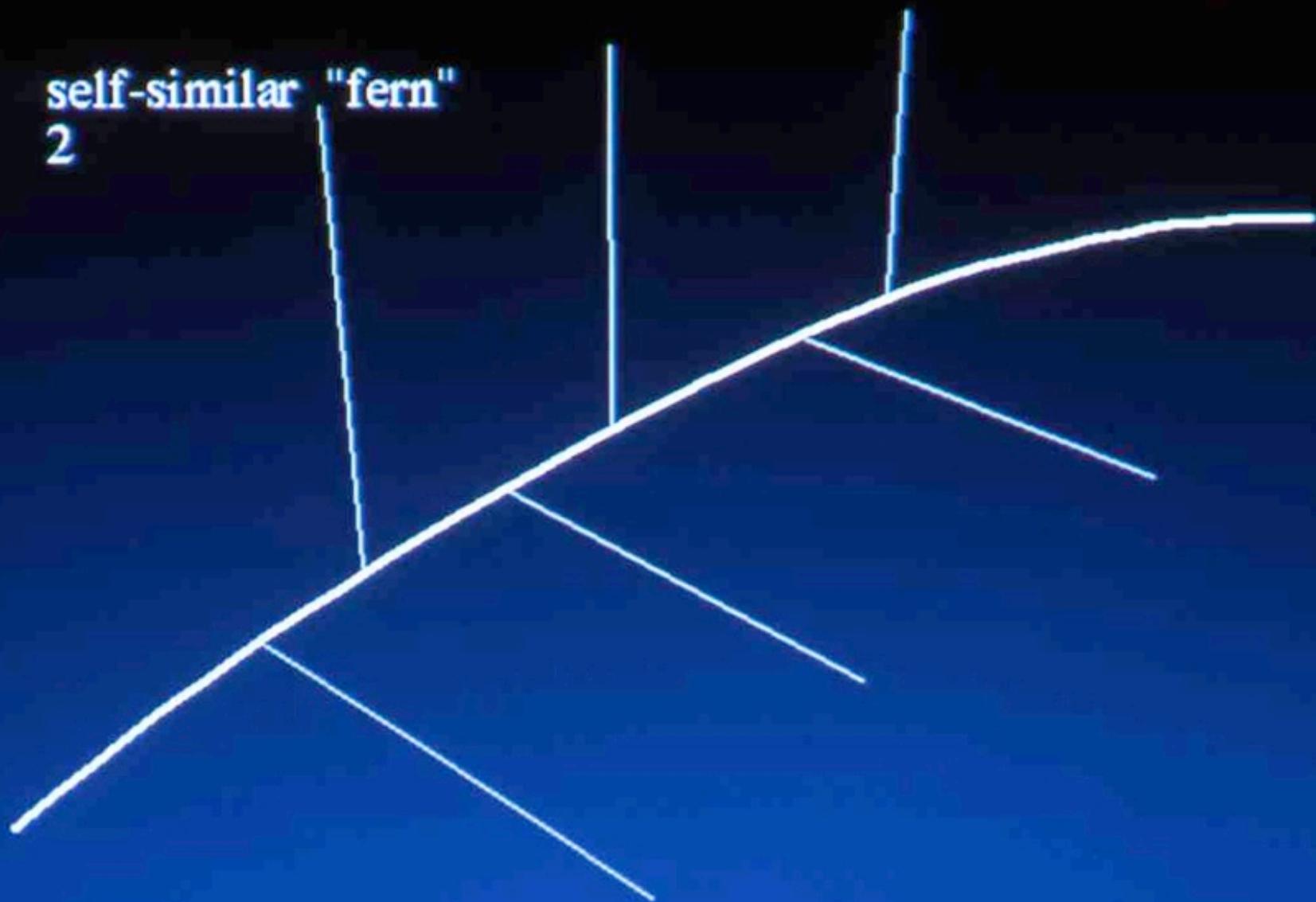
self-similar "fern"

1

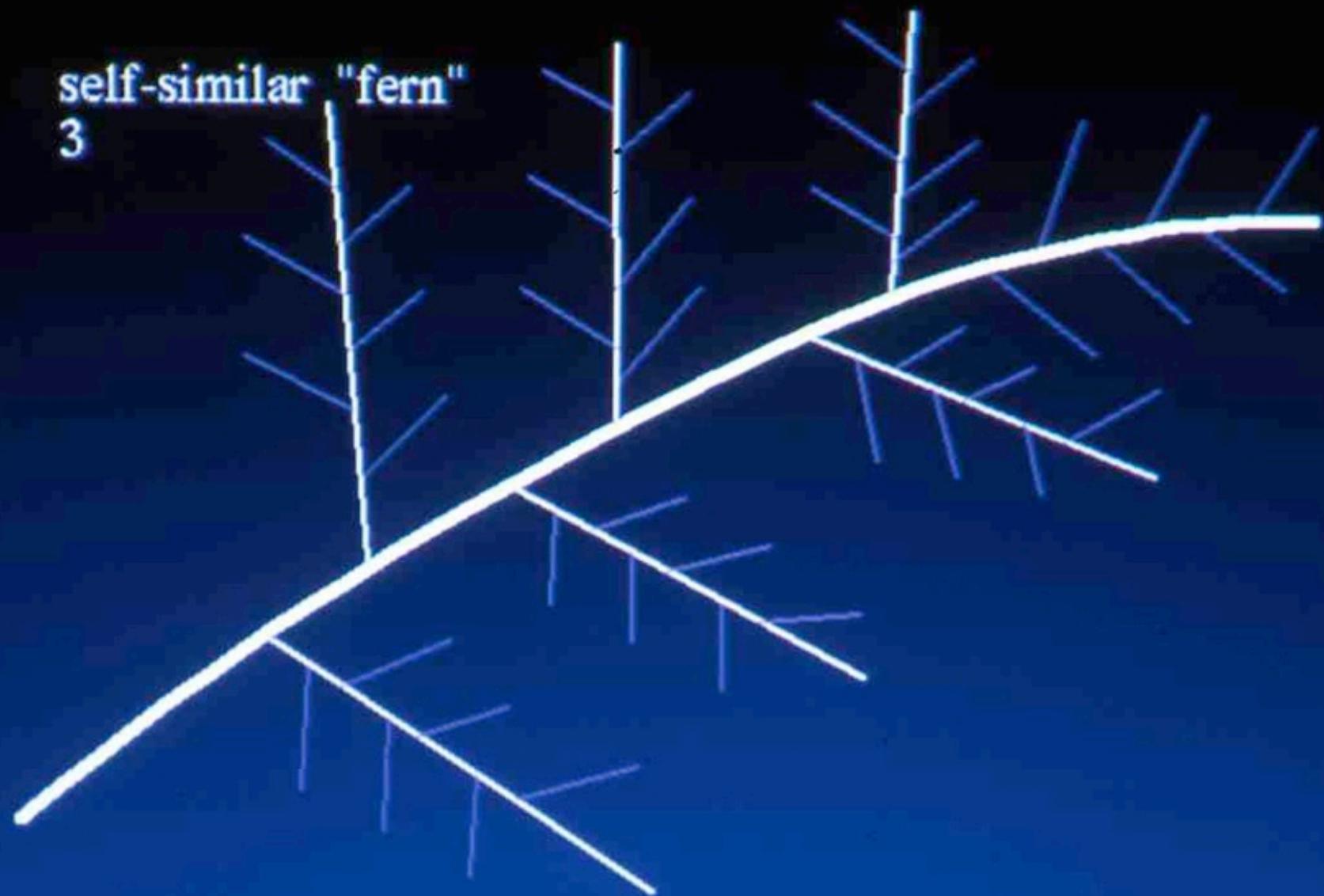


self-similar "fern"

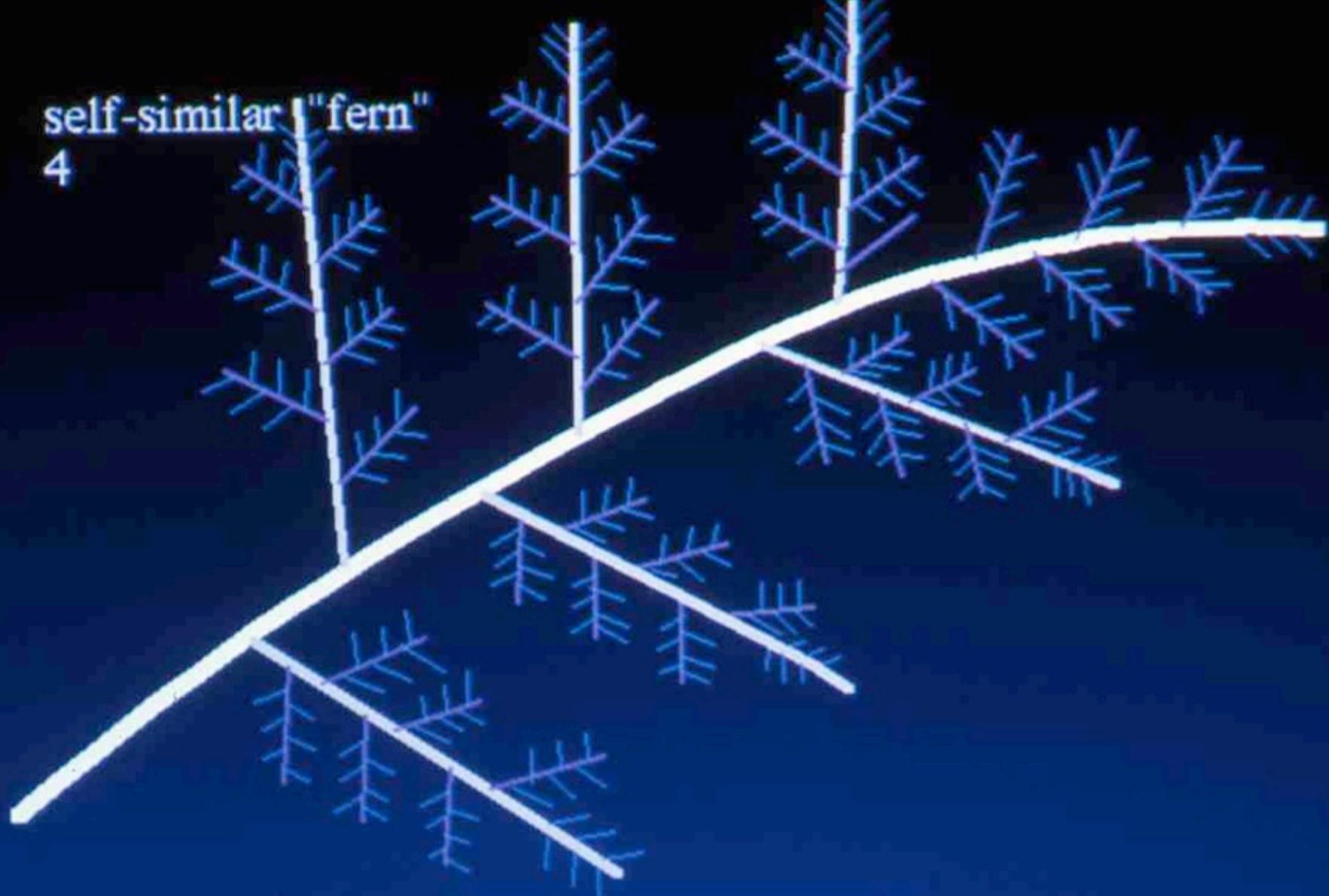
2



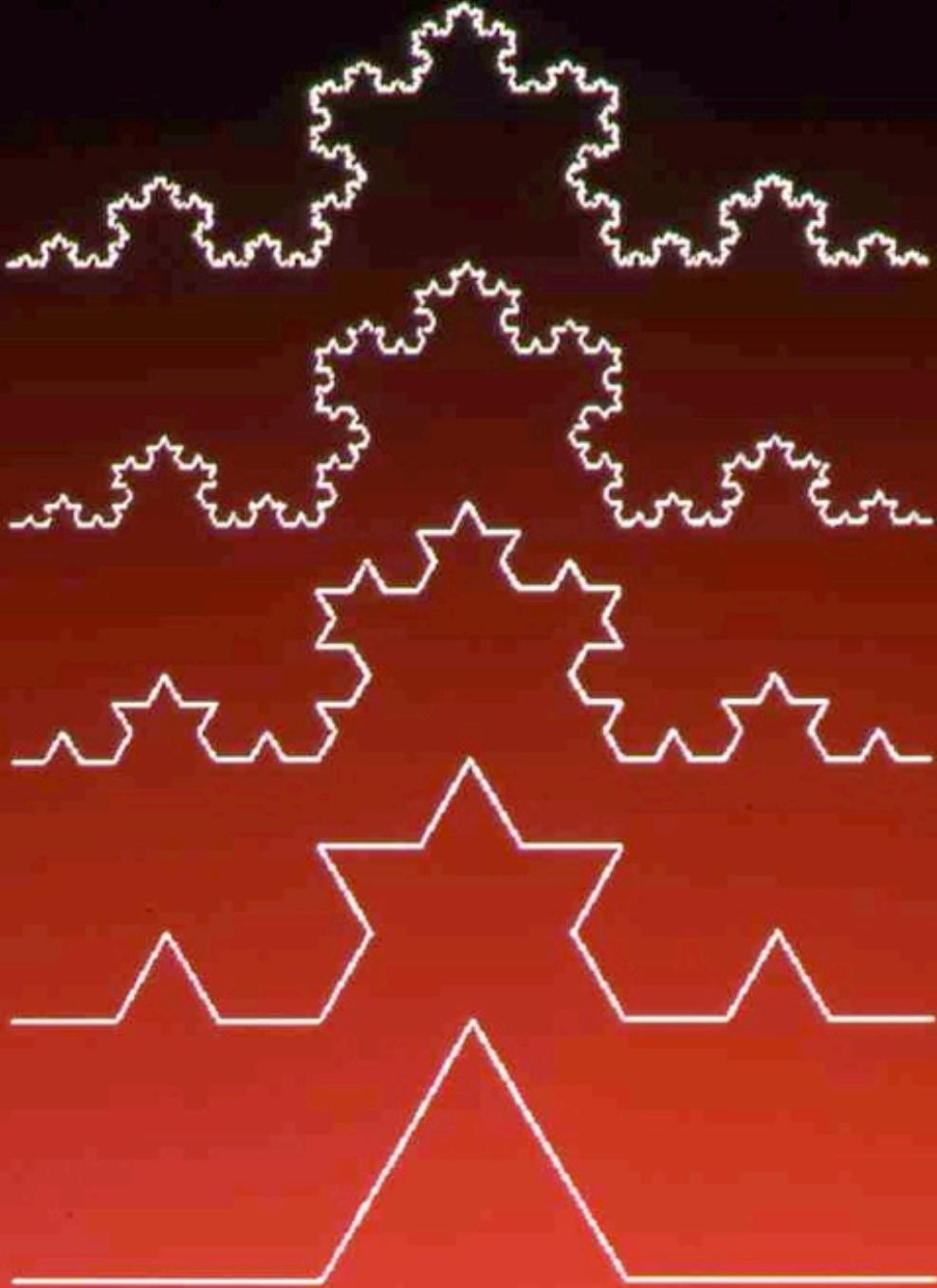
self-similar "fern"
3



self-similar
"fern"
4







courbe
von Koch 1905

$$D = \log 4 / \log 3$$

arbre botanique
structure ramifiée

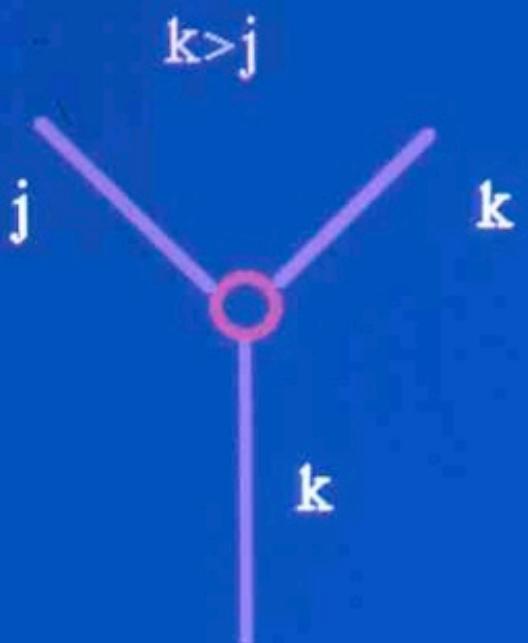


**arbre binaire
combinatoire
(topologique)**

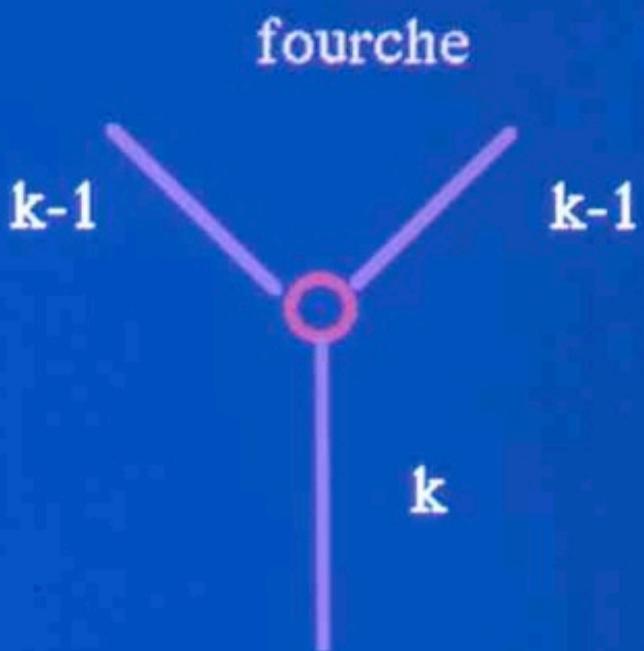


matrice de ramifications

Biordre d'un branchement d'ordre k



biordre (k,j)



biordre $(k-1,k-1)$

Relation

forme
&
matrice de ramification



2 : 0 10000
3 : 0 0 10000
4 : 0 0 0 10000
5 : 0 0 0 0 10000
6 : 0 0 0 0 0 10000
7 : 0 0 0 0 0 0 10000
8 : 0 0 0 0 0 0 0 10000
9 : 0 0 0 0 0 0 0 0 10000
10: 0 0 0 0 0 0 0 0 0 10000
11: 0 0 0 0 0 0 0 0 0 0 10000

A\$A

Ramification matrix

self-similar fern

$1-1/p$	$1/p$				
0	$1-1/p$	$1/p$			
0	0	$1-1/p$	$1/p$		
0	0	0	$1-1/p$	$1/p$	

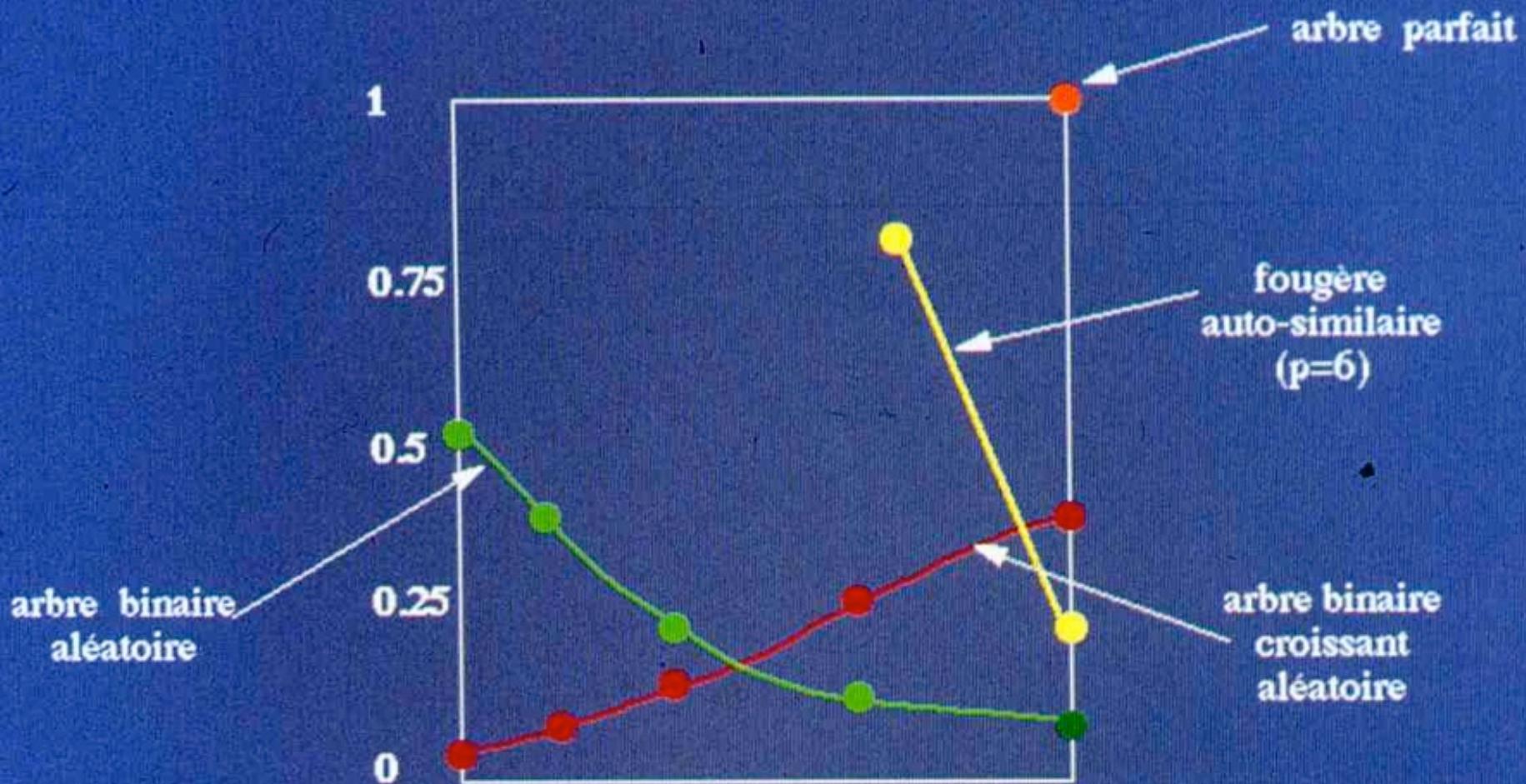
Arbre binaire aléatoire
en 3D



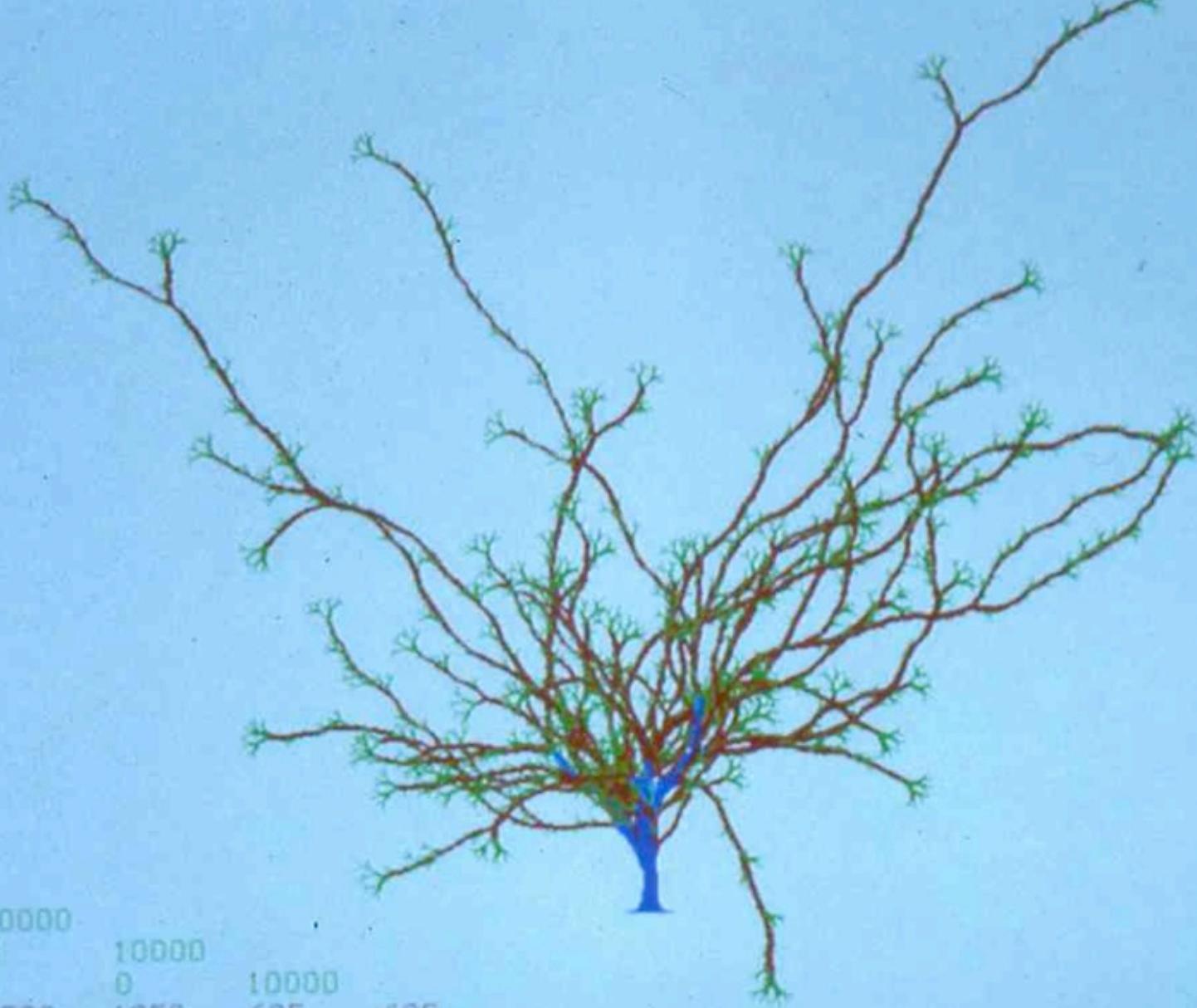
A\$A



2 : 4000	6000									
3 : 2000	3000	5000								
4 : 1000	2000	3000	4000							
5 : 500	1000	2000	3000	3500						
6 : 250	500	1000	2000	3000	3250					
7 : 125	250	500	1000	2000	3000	3125				
8 : 63	125	250	500	1000	2000	3000	3062			
9 : 31	63	125	250	500	1000	2000	3000	3031		
10: 15	31	63	125	250	500	1000	2000	3000	3016	
11: 7	15	31	63	125	250	500	1000	2000	3000	3024



matrices de ramification auto-similaires

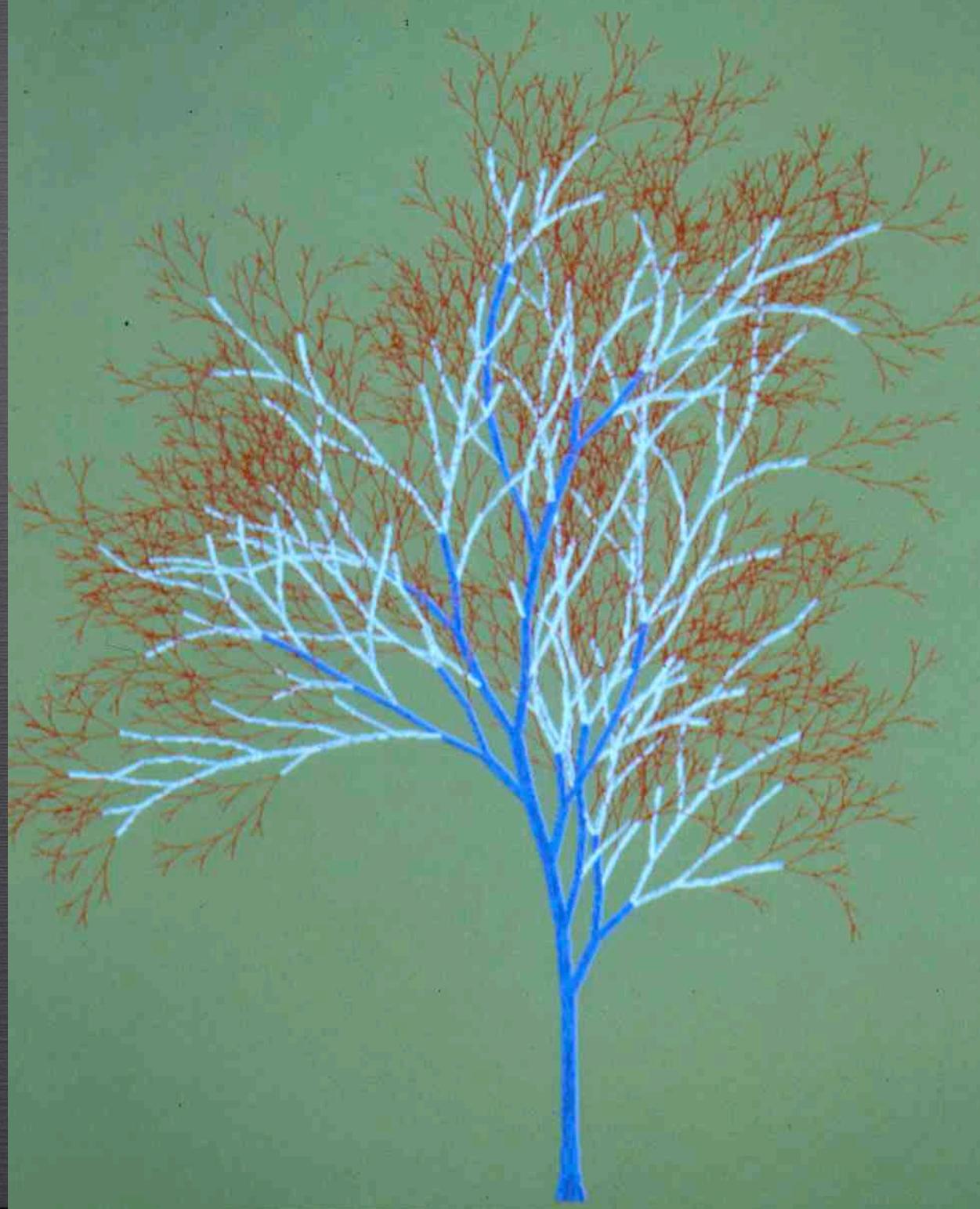


Mélange de trois matrice



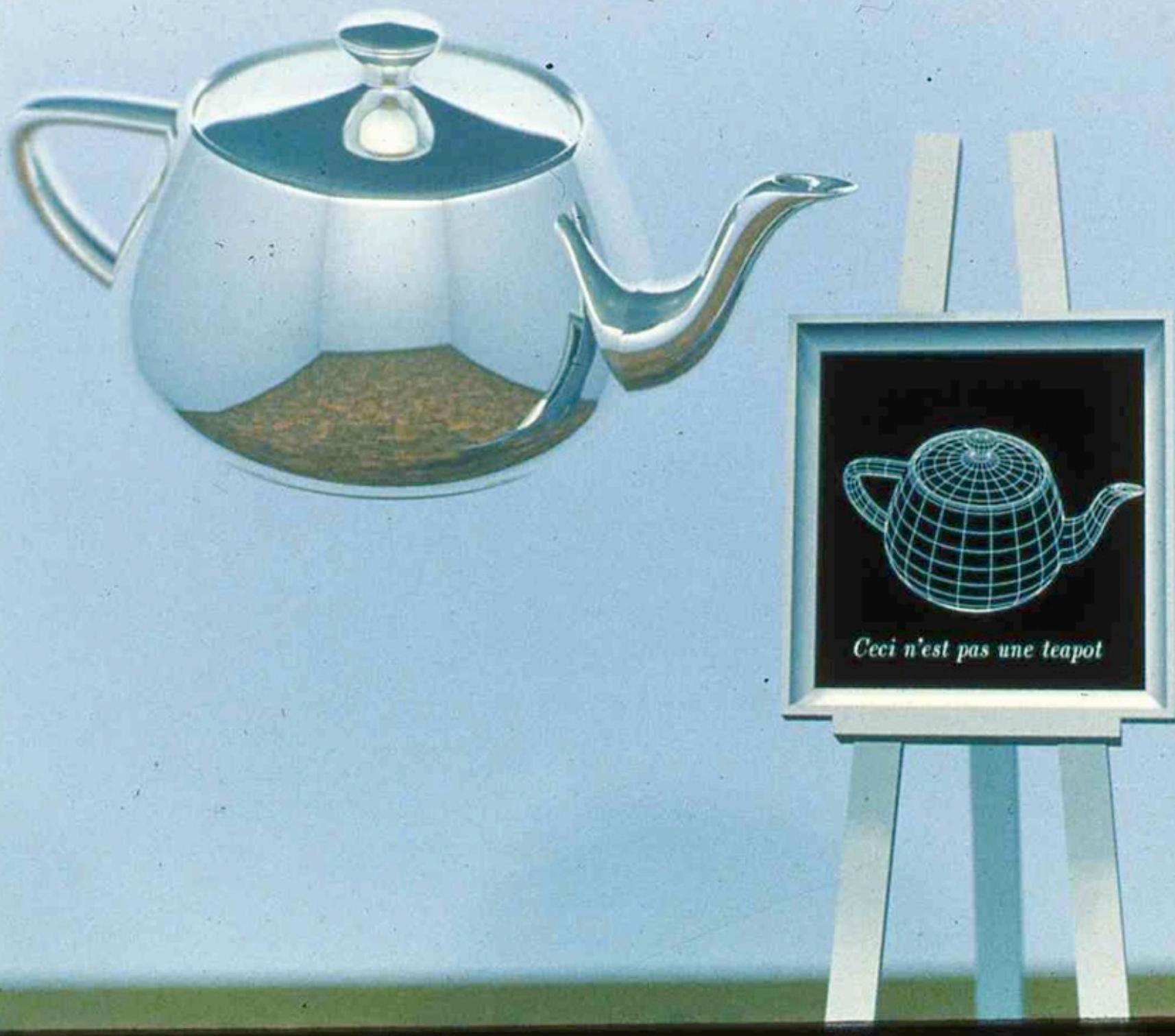
Néatoire

Algénat



8

Images de
synthèse
d'arbres



Ceci n'est pas une teapot







Atelier de Synthèse d' Images d' Arbres

CNRS
LaBRI Bordeaux
& LIB Besançon

ASA

Didier Arquès
Georges Eyrolles
Nicolas Janey
Xavier Gérard Viennot

Génération de l'arbre combinatoire

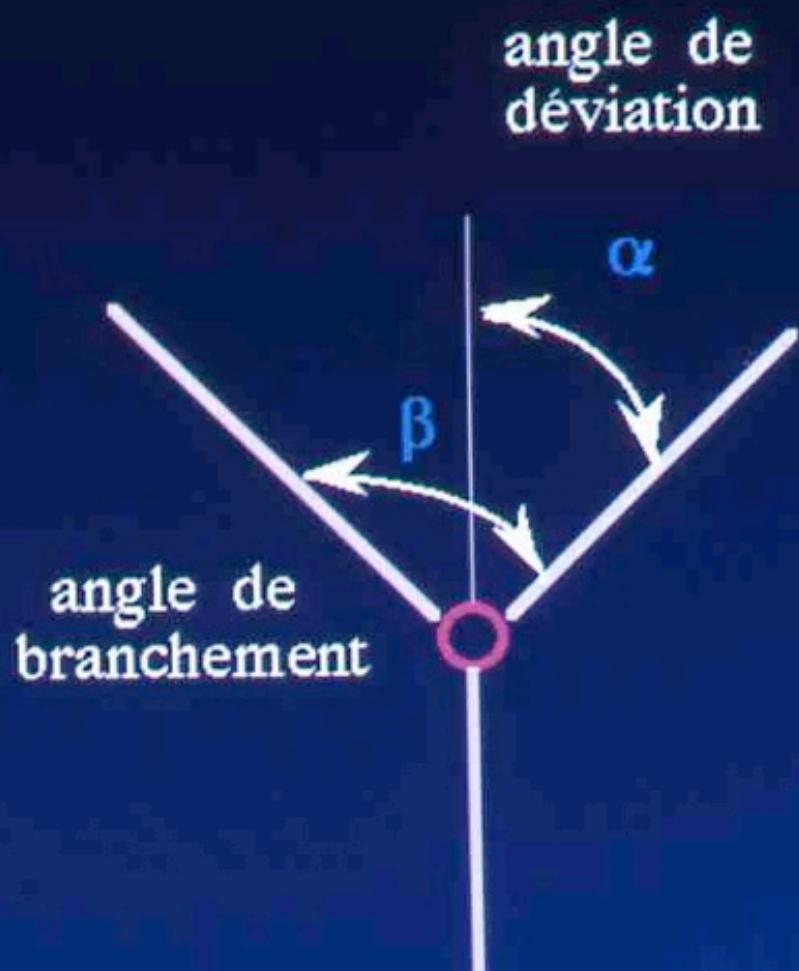
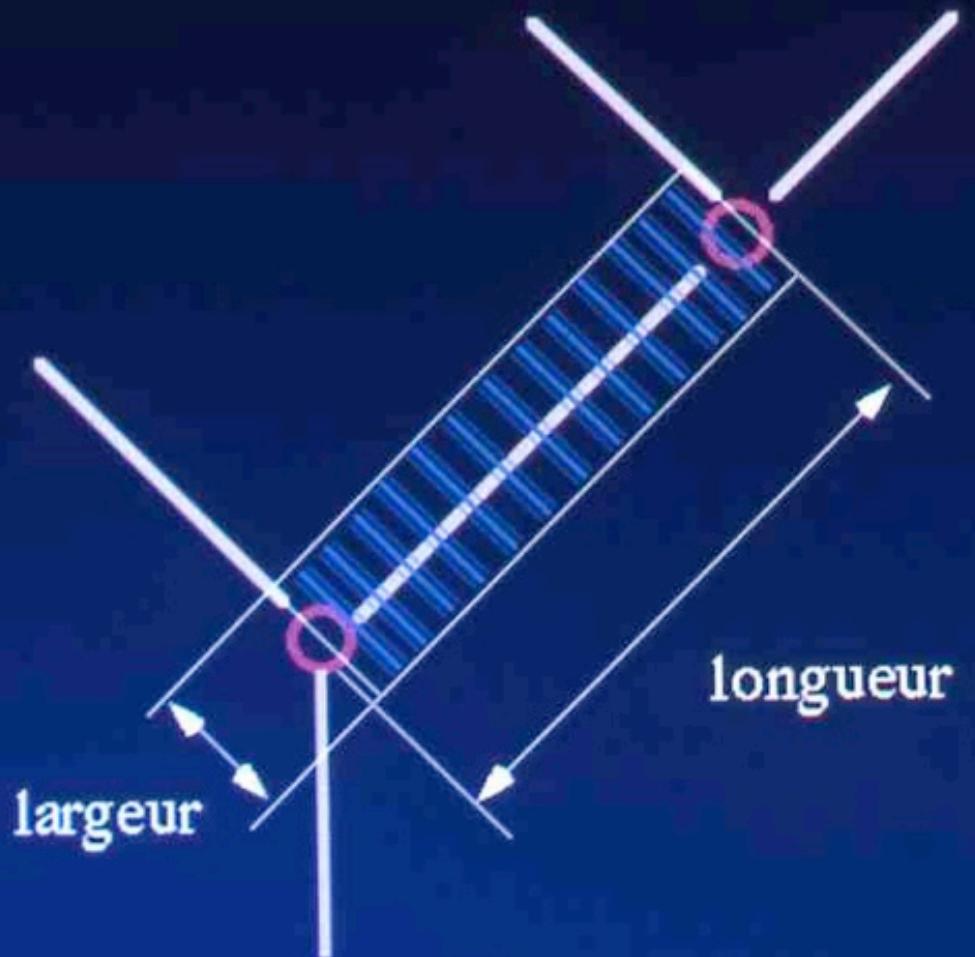
choix de la matrice de ramification

Génération de l'arbre géométrique

- largeur $w(k)$
 - longueur $L(k)$
- } fonction de l'ordre
-
- angle de déviation
 - angle de branchement
- } fonction du biordre

dessin en 2D

Segments
d'un arbre binaire



Mélange
déterminisme
aléatoire

matrice de ramification
lois géométriques

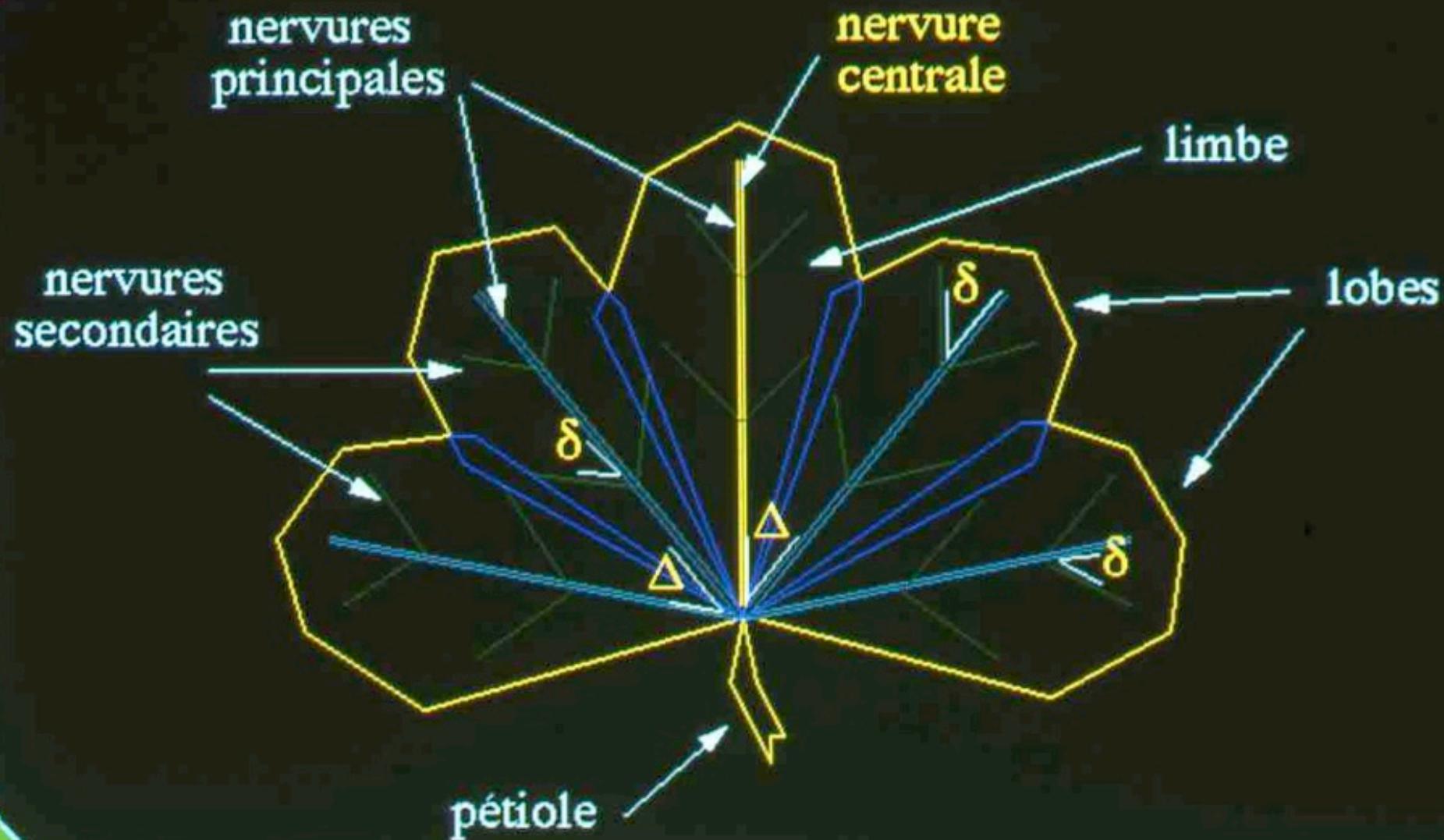
processus stochastiques

La forme finale est implicitement contenue dans son histoire

D'Arcy Thompson "On growth and form"

Ici : séparation forme et histoire

Géométrie d'une feuille









ASA



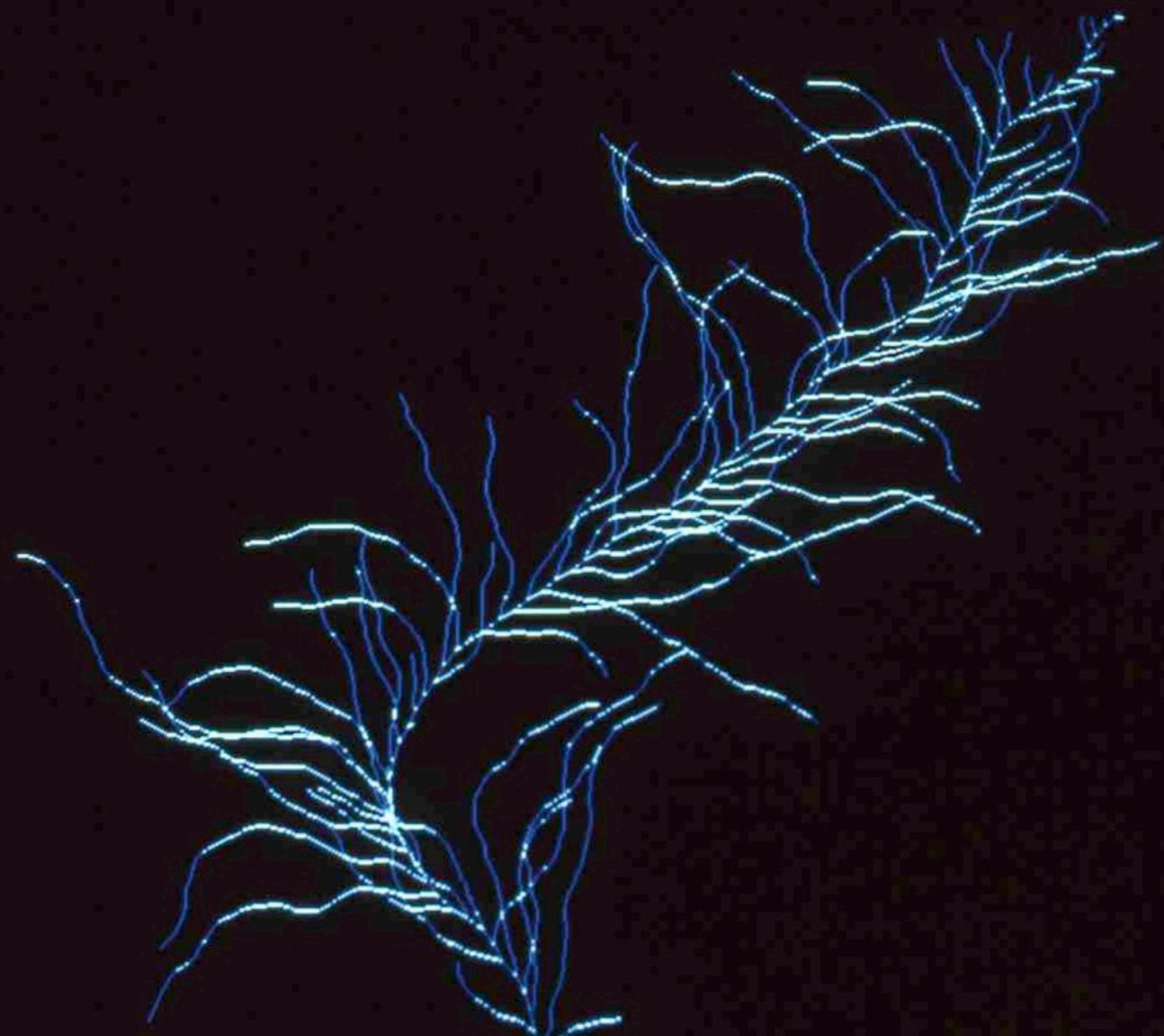


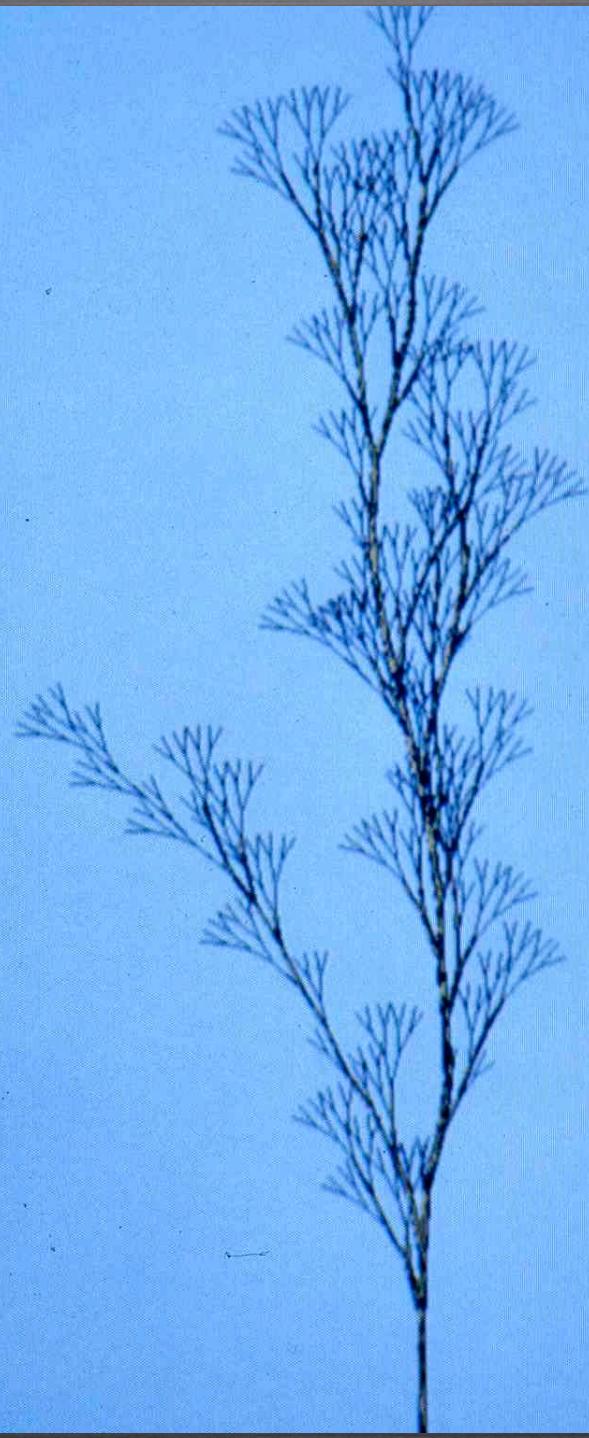


A\$A



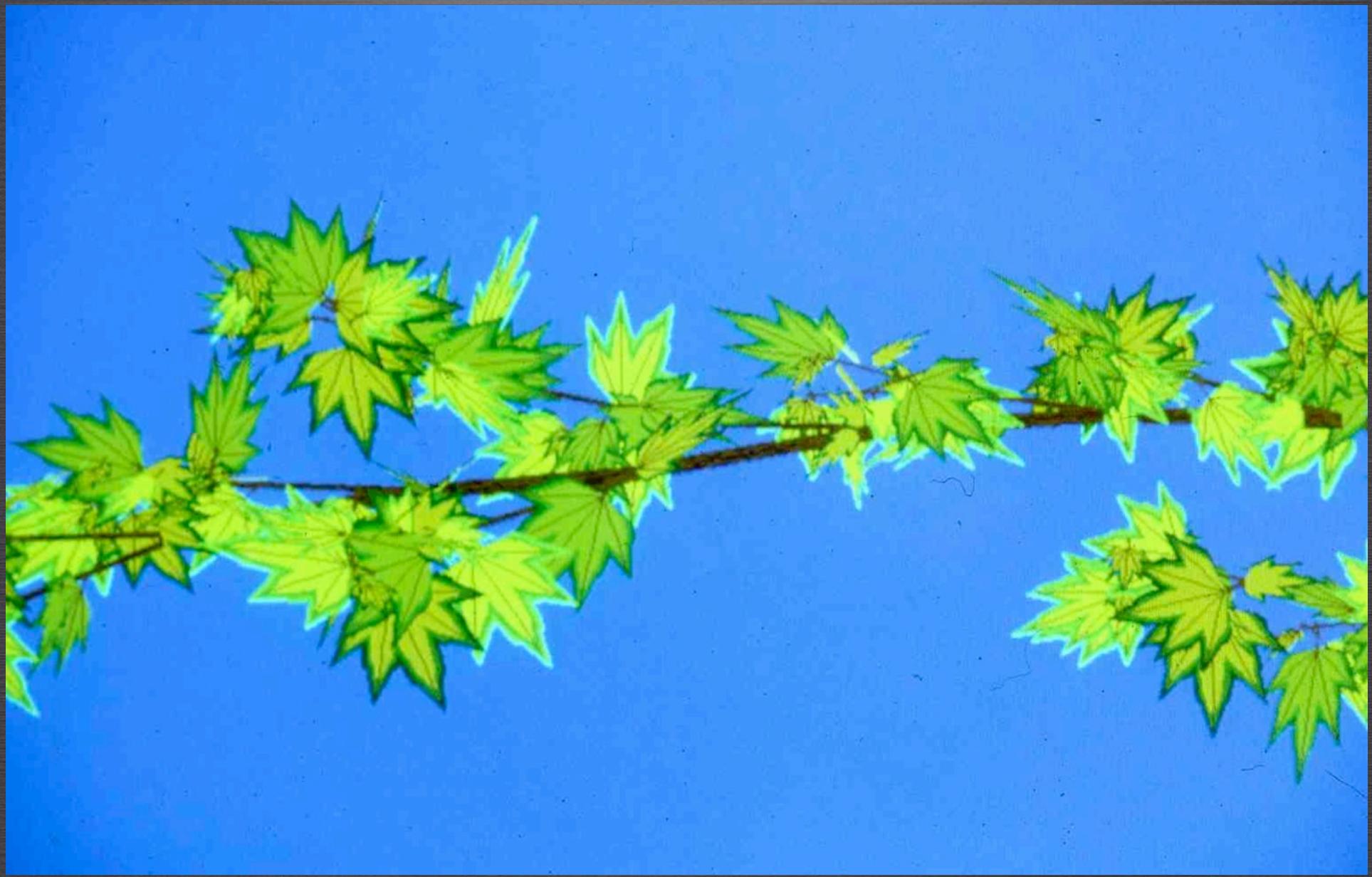


















Lib











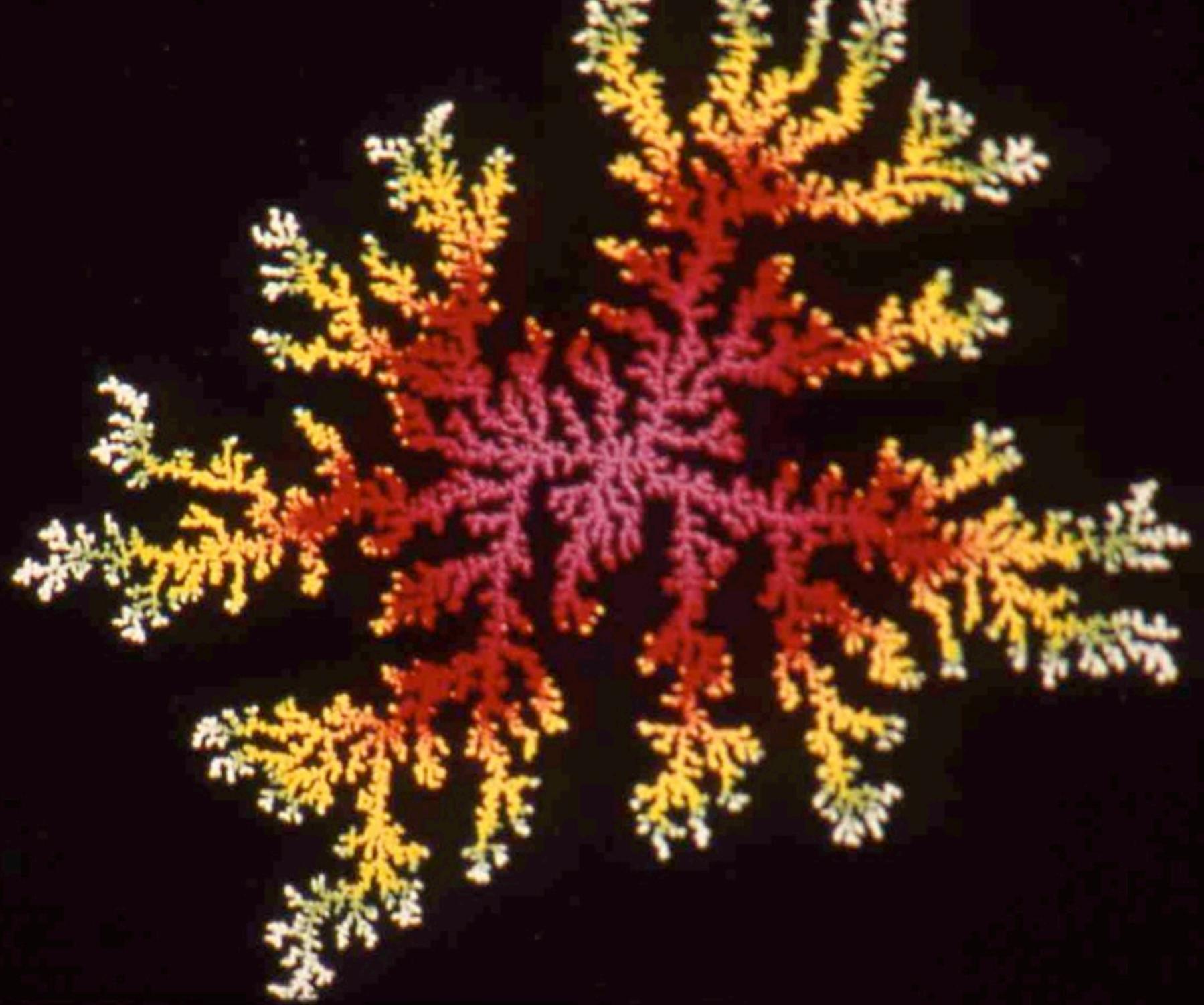




9

Matrices
de
ramification
en
physique







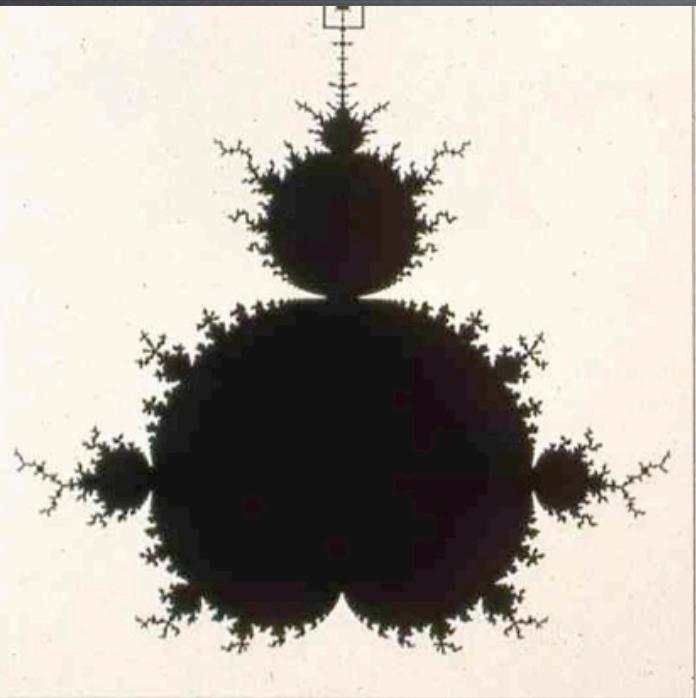
10

Systèmes dynamiques

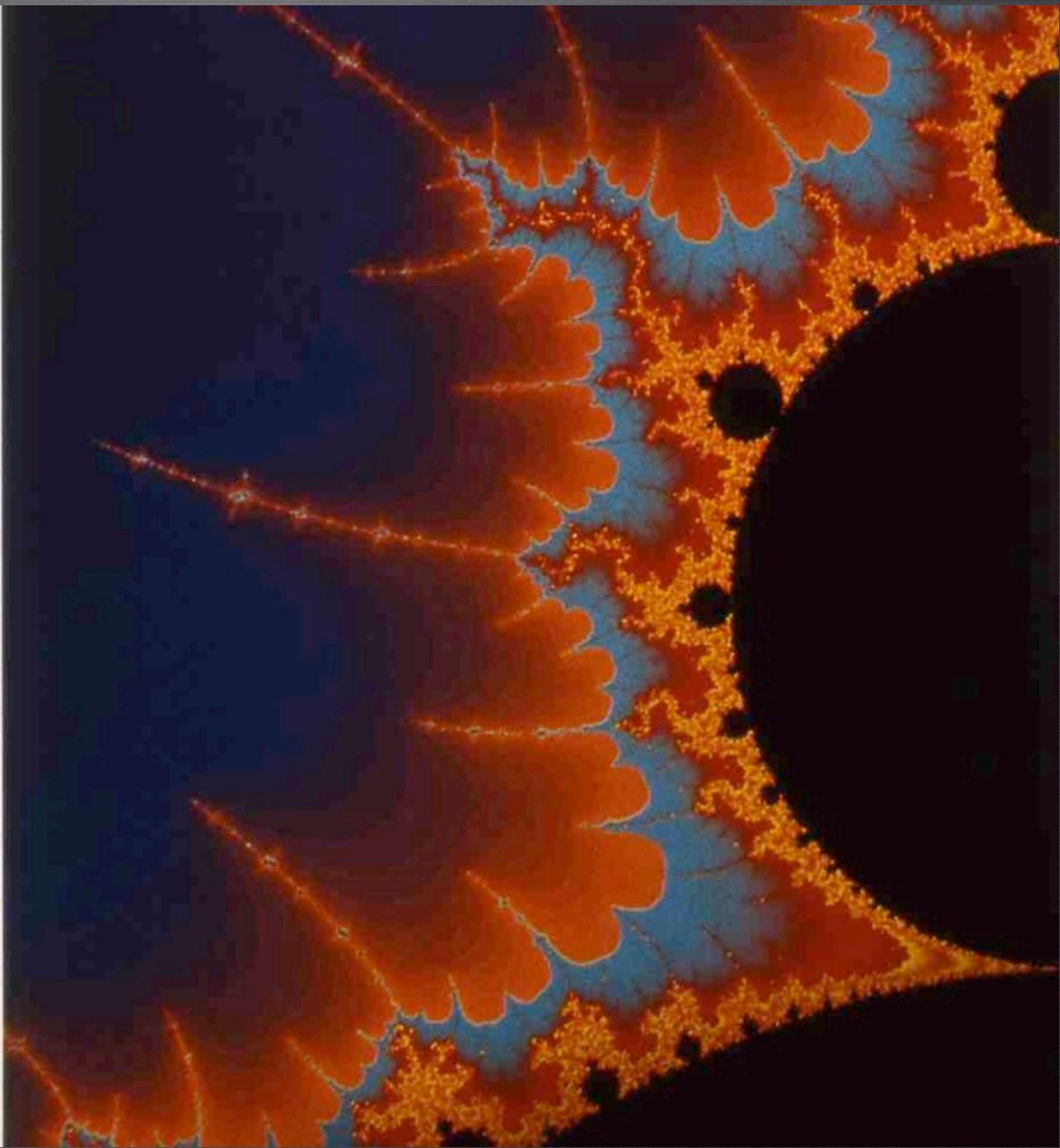
Dynamical systems

quadratic recurrence
 $f(z) = z^2 + c$

0



35dM





$$P_0(z) = z^2$$

$$P_1(z) = P_0^2(z) + c$$

.....

$$P_{n+1}(z) = P_n^2(z) + c$$

.....

$P_n(z)$ polynomial
of degree 2^n

subsequence of a
sequence $\{P_i(z)\}_{i \geq 0}$ of
orthogonal polynomials

Combinatorial interpretation
of the moments μ_n with
Horton-Strahler combinatorics

by Michael S. Shantz, Ph.D.
University of Texas at Austin

Classification of Galactograms
with ramification matrices

P. Bakic, M. Albert , A. Maidment
(2003)

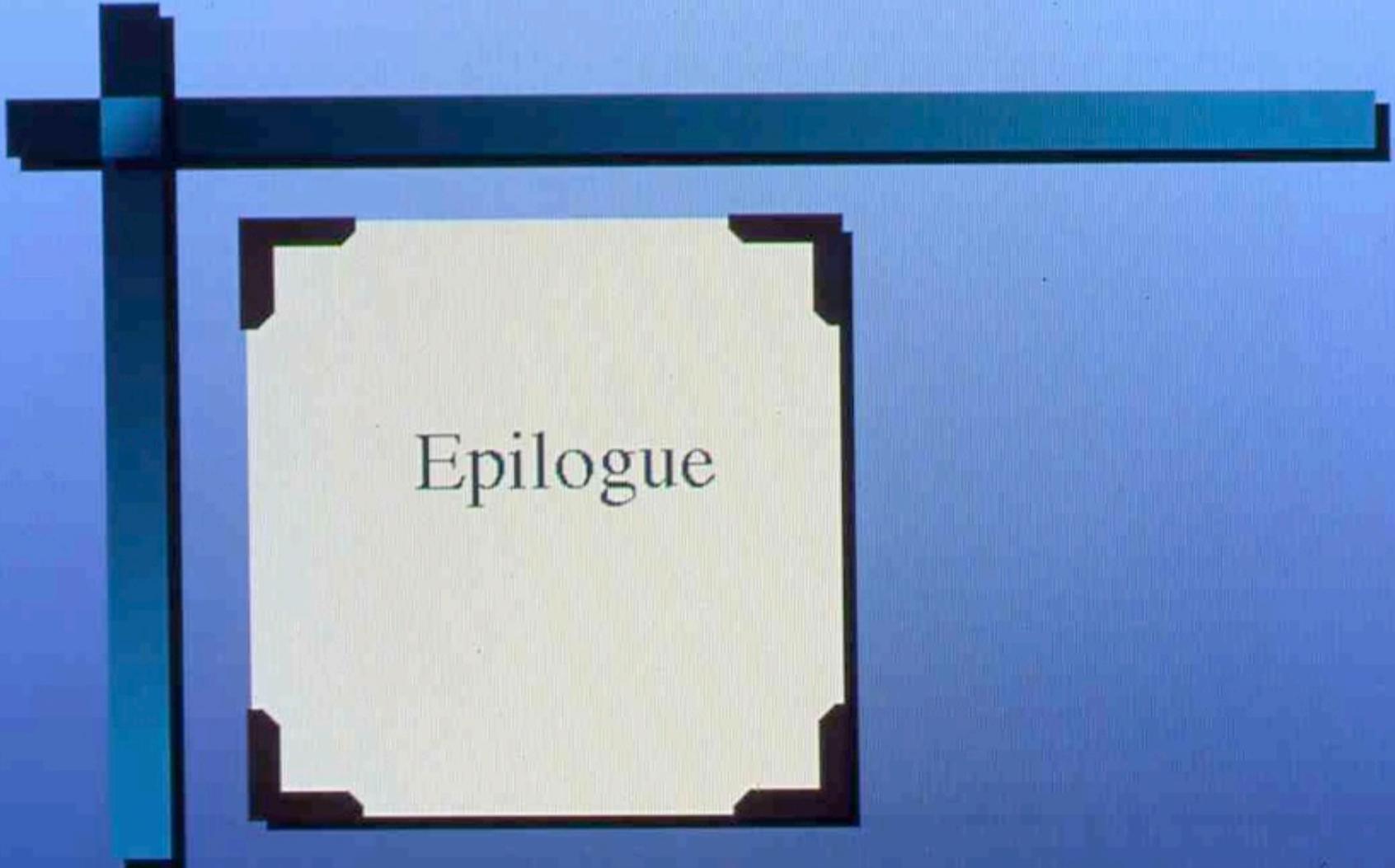
Digital mammography

• Visualisation de l'information
très grands graphes

D. Auber, M. Delest
Y. Chnicota, G. McLanahan, J.M. Fedou

analyse de Horton -
Strahler





Epilogue





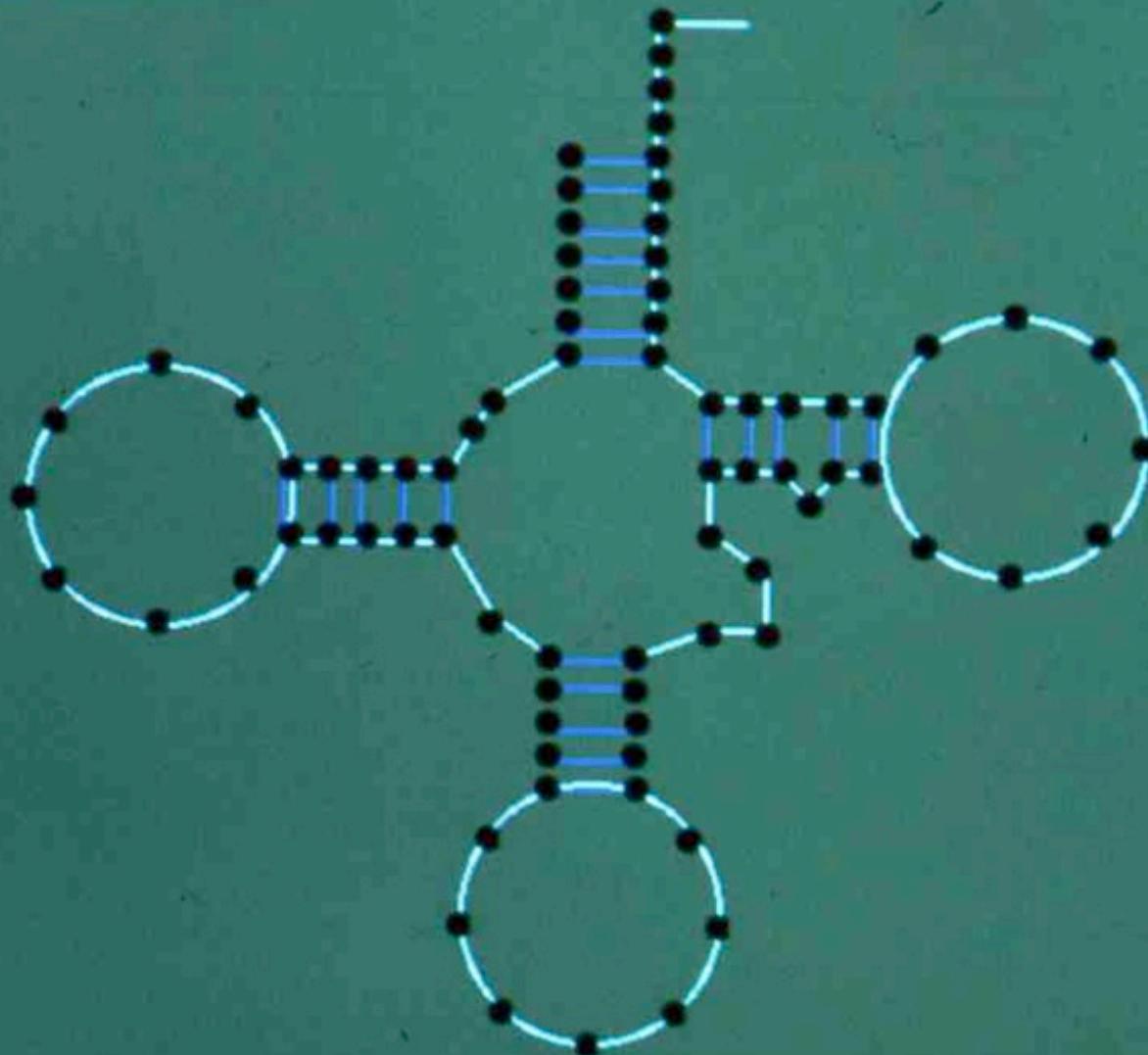


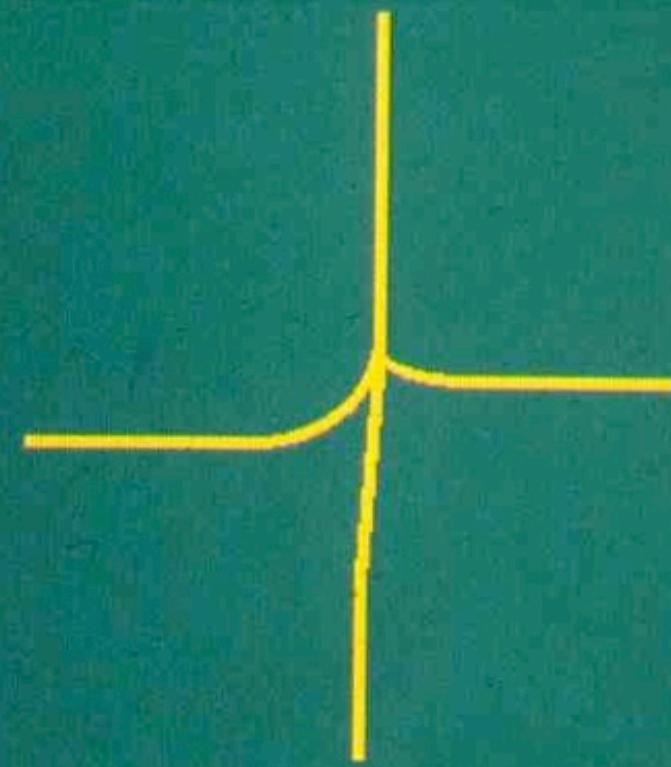


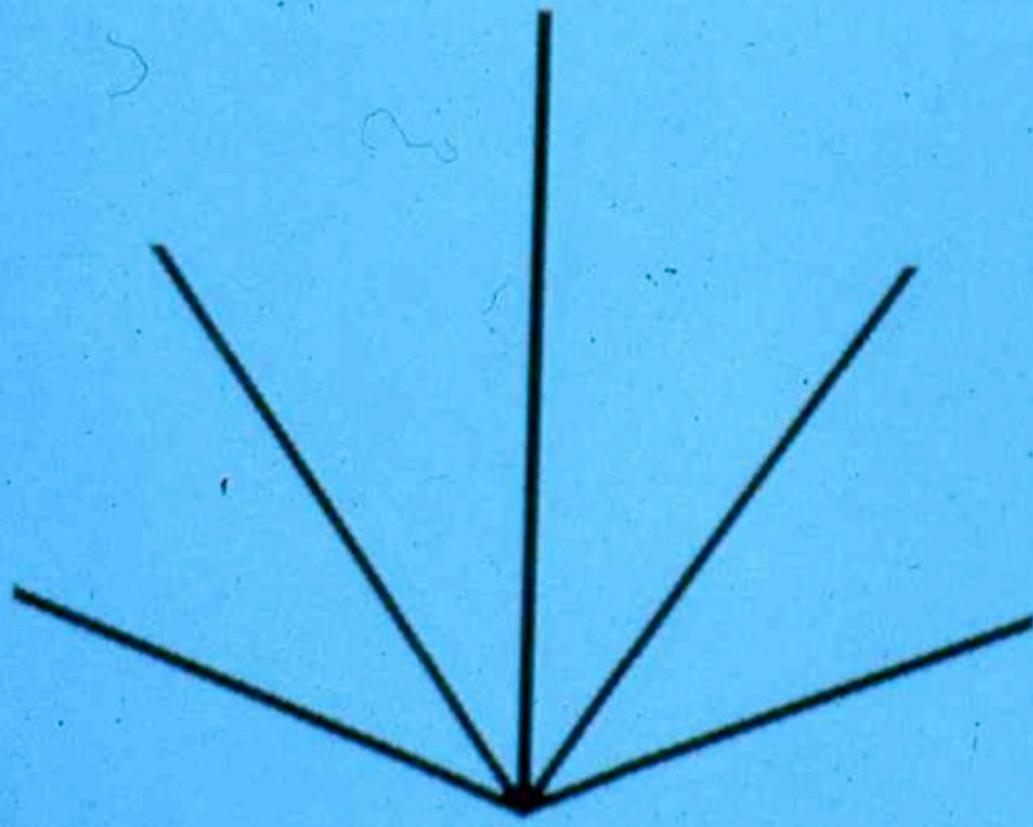
Trees everywhere

tARN^{Phe}

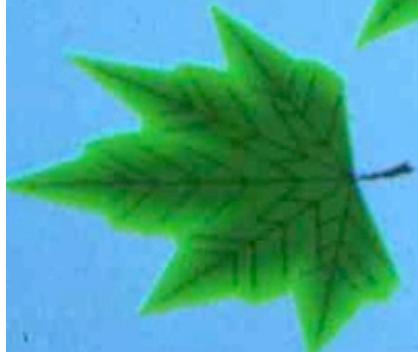
Phenylalanine

















FIN

**Auprès de mon arbre,
je vivais heureux ...**

J'aurais jamais dû m'éloigner d'mon arbre.

Georges Brassens

